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On DRC-covering of $\lambda K_t(n)$ by cycles

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Abstract. This paper considers the cycle covering of complete multipartite graphs motivated by the design of survivable WDM networks, where the requests are routed on sub-networks which are protected independently from each other. The problem can be stated as follows: for a given graph *G*, find a cycle covering of the edge set of $\lambda K_t(n)$, where $V(K_t(n))=V(G)$, such that each cycle in the covering satisfies the disjoint routing constraint (DRC). Here we consider the case where $G=C_{tn}$, a ring of size *tn* and we want to minimize the number of cycles $\rho(n^t, \lambda)$ in the covering. For the problem, we give the lower bound of $\rho(n^t, \lambda)$, and obtain the optimal solutions when *n* is even or *n* is odd and both λ and *t* are even.

Keywords: $K_t(n)$; DRC-covering; cycle; WDM network

1. Introduction

Wavelength division multiplexing (WDM) technology has the potential to satisfy the ever-increasing bandwidth needs of network users on a sustained basis. Wavelength division multiplexing is now being widely used for expanding capacity in optical networks. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths, thus multiple channels can be created within a single fiber. There are two basic architectures used in WDM networks: ring and mesh. The majority of optical networks in operation today have been built based on the ring architecture. The reader is referred to [9-14] for the relevant work.

Here we consider a covering problem arising from the decomposition of a survivable WDM network, where the communication requests are routed on sub-networks which are protected independently from each other. We model the WDM network by a graph, called the physical graph and denoted by *G*. The vertices of the graph represent the optical switches and the edges the fiber-optics links. In fact, *G* is an oriented symmetric multigraph; indeed each time there is a fiber optic from a node *x* to a node *y* there is also the opposite one. We will consider that the graph G is either an undirected cycle of length tn, denoted by C_{tn}, or the symmetric directed cycle C_{tn}^{*}.

Routing a request over G consists in finding a path over G between the pair of nodes communicating in the request. The protection problem we consider consists in covering the family of requests by some small subcycles. Indeed, on the cycle we use half of the capacity for the demands, and in case of failure we reroute the traffic through the failed link via the remaining part of the cycle using the other half of the capacity. It will be interesting to get very small cycles as subnetworks as they are easier to manage and less costly to reroute. For the reason, one constraint that each cycle formed by some requests must be routed vertex disjointly over G, or this is the same as saying that we can find a set of vertex disjoint paths corresponding to the set of requests over a cycle. We call this property the disjoint routing constraint (DRC).

We can model all the requests as the edge set of a logical graph I undirected or not. The vertices represent the nodes of the physical graph and the edges correspond to the requests between these nodes.

Problem 1.1. Find a cycle partition or a cycle covering satisfying DRC property of the edges of I with an associated routing over G such that the number of cycles is minimized.

Our aim is to minimize the cost of the network. When $G=C_{tn}$, it corresponds to minimize the number of cycles in the covering. In summary, we want to find the minimum number of cycles in a DRC-covering of I relatively to the cycle C_{tn}. As a variation of the problem, we can also add some restriction to the cycles in the covering, for example, we can consider the case when the size of the cycles is uniform or is bounded. In [1], Bermond et al. discussed the problem of DRC-covering for the logical graph I is the complete graph K_n (or the symmetric complete digraph K_n^{*}), and the physical graph G is a cycle. Bermond and Yu [2] extended the results to G be a torus (instead of cycle). Liang and Han [3] discussed the problem of DRC-covering for the logical graph I is λK_n and $\lambda K_{n,n}$ (or λK_n^{*} and $\lambda K_{n,n}^{*}$). Recently, the survey [15] lists the results on DRC cycle coverings. In this paper, we will discuss the case for the logical graph I is $\lambda K_t(n)$.

2. Definitions and notation

Let Z be a ring of integers and Z_m the residue class group modulo m with residue classes $\{0,1,\ldots,m-1\}$. In what follows, the notations, $[a,b]=\{x\in Z \mid a\leqslant x \leqslant b\}$, $[a,b]_k=\{x\in Z \mid a\leqslant x\leqslant b, x\equiv a \pmod k\}$ for $a,b\in Z$ and $\lceil x\rceil=min\{y \mid y\in Z, y\geqslant x\}$ are used frequently.

We use the notation of graph theory so that K_n, $K_t(n)$ and C_m will, respectively, denote the complete graph on n vertices, the complete t-partite graph with parts of sizes n, and the m-cycle. λ copies of $K_t(n)$ is denoted by $\lambda K_t(n)$. Let the vertices of the physical graph C_{tn} be labeled with integers modulo tn, represented by the set $\{0,1,2,...,tn-1\}$. For that, let us arrange the vertices of C_{tn} in the following order: $0,1,2,...,tn-1$. Let $X_i=\{x \mid x\in Z_{tn}, x\equiv i \pmod t\}$, $i\in[0,t-1]$ be parts of the logical graph $\lambda K_t(n)$. Let us give with a few basic definitions again. Let Ω be a set of cycles, and $H=\lambda K_t(n)$. A *Ω-decomposition* (*Ω-covering*, resp.) of H, denoted by $GD(H, \Omega)$ ($CD(H, \Omega)$, resp.), is a pair (X,B) where X is the vertex set of H and B is a collection of subgraphs of $K_t(n)$, such that each subgraph is isomorphic to some cycle in Ω and each edge in $K_t(n)$ is joined in exactly (at least, resp.) λ subgraphs of B. A Ω-covering is said to be *minimum*, denoted by $MCD(H, \Omega)$, if no other such Ω-covering has fewer subgraphs. The number of subgraphs in a minimum Ω-covering is called the *covering number*, denoted by $c(H, \Omega)$.

If a Ω-decomposition (minimum Ω-covering, resp.) satisfies DRC property, we call it is a DRC-decomposition (DRC-covering, resp.). In the following, we denote $\rho(n^t,\lambda)$ the minimum number of cycles needed in such a DRC-covering of $\lambda K_t(n)$ and similarly we define $\rho_k(n^t,\lambda)$ for the case when the cycle size is restricted to be k. It is easy to obtain the following result.

Proposition 2.1. $\rho(n^t,\lambda) \geqslant c(\lambda K_t(n), \Omega)$, and $\rho_k(n^t,\lambda) \geqslant c(\lambda K_t(n), \Omega)$ if $\Omega=\{C_k\}$.

It is easy to see that a cycle C_k satisfies DRC if and only if its vertices can be ordered cyclically modulo tn, that is if the vertices can be written $(a_1,a_2,\dots,a_k)$ with $0\leqslant a_1\leqslant a_2\leqslant\cdots\leqslant a_k\leqslant tn-1$. For convenience sake, we denote the C_k by $(a_1,a_2,\dots,a_k,a_1)$.

3. The main results

In this section, our main aim will be to find the minimum number $\rho(n^t,\lambda)$ of cycles in a DRC-covering of $\lambda K_t(n)$.

Theorem 3.1. Let n, t and λ be positive integers. The lower bound of $\rho(n^t,\lambda)$ is as follows.

1. When t and n are odd, $\rho(n^t,\lambda) \geqslant \lambda(t-1)(tn^2+1)/8$.
2. When t is even and n is odd, $\rho(n^t,\lambda) \geqslant [\lambda t(tn^2-n^2+1)+4]/8$ if λ is odd, $\rho(n^t,\lambda) \geqslant \lambda t(tn^2-n^2+1)/8$ if λ is even.
3. When n is even, $\rho(n^t,\lambda) \geqslant \lambda t(t-1)n^2/8$.

Proof. Let C^j, $1 \leqslant j \leqslant \rho(n^t, \lambda)$, be the cycles of a DRC-covering of $\lambda K_t(n)$, the disjoint routing property implies that the vertices of any C^j are cyclically ordered modulo *tn*. Thus the C^j can be written ($a_1^j, a_2^j, \ldots, a_{k_j}^j, a_1^j$) with $0 \leqslant a_1^j \leqslant a_2^j \leqslant \ldots \leqslant a_{k_j}^j \leqslant tn-1$.

Let $\delta_i^j = a_{i+1}^j - a_i^j$, $1 \leqslant i \leqslant k_j-1$, and $\delta_{k_j}^j = tn + a_1^j - a_{k_j}^j$. The disjoint routing property implies $\sum_i \delta_i^j = tn$. For an edge *xy* of $\lambda K_t(n)$ with *x*<*y*, we call difference of the edge the value *y-x* if $y-x \leqslant tn/2$ or *x+tn-y* otherwise.

Case 1. When *t* and *n* are odd

The covering must contains $t\lambda n$ edges of difference *d*, $d \in [1,(tn-1)/2]-[t,(tn-1)/2]_t$. We have

$$\sum_{i,j} \delta_i^j \geq t\lambda n\Big(\sum_{d=1}^{(tn-1)/2} d - t\sum_{d=1}^{(n-1)/2} d\Big) = t\lambda n\Big[\tfrac{t^2n^2-1}{8} - \tfrac{t}{2}\cdot\tfrac{n-1}{2}\cdot\tfrac{n+1}{2}\Big] = \tfrac{t\lambda n(t-1)(tn^2+1)}{8}.$$

If the covering contains $\rho(n^t, \lambda)$ cycles, we obtain

$$nt\,\rho(n^t, \lambda) \geqslant t\lambda n(t-1)(tn^2+1)/8.$$

Therefore, $\rho(n^t, \lambda) \geqslant \lambda(t-1)(tn^2+1)/8$.

Case 2. When *t* is even and *n* is odd

(i) If λ is odd, the covering must contains $t\lambda n$ edges of difference *d*, $d \in [1,(tn-2)/2]-[t,(tn-2)/2]_t$, and contain $t\lambda n/2$ edges of difference *tn/2*. Furthermore, since the degree of the nodes in $\lambda K_t(n)$ is odd, and the degree of the nodes of a cycle is even, the covering must contains extra edges. Thus, there are at least *nt/2* extra edges of difference at least *1* in the covering. Consequently,

$$\sum_{i,j} \delta_i^j \geq \tfrac{tn}{2} + t\lambda n\Big(\sum_{d=1}^{(tn-2)/2} d - t\sum_{d=1}^{(n-1)/2} d\Big) + \tfrac{t\lambda n}{2}\cdot\tfrac{tn}{2}$$
$$= \tfrac{tn}{2} + t\lambda n\Big[\tfrac{tn(tn-2)}{8} - \tfrac{t(n^2-1)}{8}\Big] + \tfrac{\lambda t^2 n^2}{4}$$
$$= \tfrac{tn[t\lambda(tn^2-n^2+1)+4]}{8}.$$

If the covering contains $\rho(n^t, \lambda)$ cycles, we obtain

$$nt\,\rho(n^t, \lambda) \geqslant tn[\lambda t(tn^2-n^2+1)+4]/8.$$

Therefore, $\rho(n^t, \lambda) \geqslant [\lambda t(tn^2-n^2+1)+4]/8$.

(ii) If λ is even, then the covering must contains $t\lambda n$ edges of difference *d*, $d \in [1,(tn-2)/2]-[t,(tn-2)/2]_t$, and contains $t\lambda n/2$ edges of difference *tn/2*. Consequently,

$$\sum_{i,j}\delta_i^j \ge t\lambda n(\sum_{d=1}^{(tn-2)/2} d - t\sum_{d=1}^{(n-1)/2} d) + \tfrac{t\lambda n}{2}\cdot\tfrac{tn}{2}$$
$$= t\lambda n[\tfrac{tn(tn-2)}{8} - \tfrac{t(n^2-1)}{8}] + \tfrac{\lambda t^2 n^2}{4} = \tfrac{\lambda t^2 n(tn^2-n^2+1)}{8}.$$

If the covering contains $\rho(n^t,\lambda)$ cycles, we obtain

$nt\rho(n^t,\lambda) \geqslant \lambda t^2 n(tn^2-n^2+1)/8$.

Therefore, $\rho(n^t,\lambda) \geqslant \lambda t(tn^2-n^2+1)/8$.

Case 3. <u>When *n* is even</u>

The covering must contains $t\lambda n$ edges of difference *d*, $d\in[1,tn/2]-[t,tn/2]_t$. Thus,

$$\sum_{i,j}\delta_i^j \ge t\lambda n(\sum_{d=1}^{tn/2} d - t\sum_{d=1}^{n/2} d) = t\lambda n[\tfrac{tn(tn+2)}{8} - \tfrac{tn(n+2)}{8}] = \tfrac{\lambda t^2 n^3(t-1)}{8}.$$

If the covering contains $\rho(n^t,\lambda)$ cycles, we obtain

$$nt\rho(n^t,\lambda) \geqslant \lambda t^2 n^3(t-1)/8.$$

Therefore, $\rho(n^t,\lambda) \geqslant \lambda tn^2(t-1)/8$. □

Lemma 3.2. Let λ be a positive integer and Ω be a set of cycles. If there exists a DRC-decomposition of $K_{n,n}$ by Ω, then there exists a DRC-decomposition of $\lambda K_t(n)$ by Ω for any integer $t\geqslant 2$.

Proof. Let the *t* parts of $K_t(n)$ be $X_i=\{ jt+i \mid j\in[0,n-1] \}$, $i\in[0,t-1]$. Since there exists a DRC-decomposition of $K_{n,n}$ by Ω, then when $i, j\in[0,t-1]$ and $i<j$, there exists a DRC-decomposition of $K_{n,n}$ with bipartition (X_i,X_j) by Ω, and let its cycle-set be $B_{i,j}$. It is easy to verify that $\cup_{i=1}^{t-1}\cup_{j=i+1}^{t-1} B_{i,j}$ is a cycle-set of the DRC-decomposition of $K_t(n)$ by Ω. Each cycle in the DRC-decomposition of $K_t(n)$ repeats λ times, we obtain the required DRC-decomposition of $\lambda K_t(n)$. □

Theorem 3.3. Let *n* be even and λ be positive integer. Then $\rho(n^t,\lambda) = \rho_4(n^t,\lambda) = \lambda t(t-1)n^2/8$.

Proof. When $n\equiv 0 \pmod 4$, we construct a DRC-decomposition of $\lambda K_{n,n}$ as follows: $(i,i+2j-1,i+n,i+n+2j-1,i)$, $0\leqslant i\leqslant n-1$ and $1\leqslant j\leqslant n/4$, and each cycle of them repeats λ times. One can check that these cycles satisfy DRC and each edge of $\lambda K_{n,n}$ is covered by one of these cycles. Then

$$\rho_4(n^2,\lambda) = \lambda n\cdot n/4 = \lambda n^2/4.$$

When $n\equiv 2 \pmod 4$, if $n=2$, then one C_4 is $(0,1,2,3,0)$. If $n>2$, we construct a DRC-decomposition of $K_{n,n}$ as follows:

$(i,i+2j-1,i+n,i+n+2j-1,i)$, $0\leqslant i\leqslant n-1$ and $1\leqslant j\leqslant (n-2)/4$,

$(i,i+n/2,i+n,i+3n/2,i)$, $0\leqslant i\leqslant (n-2)/2$.

Note that these cycles satisfy DRC property. Each cycle of them repeats λ times, we obtain a DRC-decomposition of $\lambda K_{n,n}$. Therefore, we have

$$\rho_4(n^2,\lambda)=\lambda[n(n-2)/4+n/2]=\lambda n^2/4.$$

From Lemma 3.2, there is $\rho_4(n^t,\lambda)=\binom{t}{2}\frac{\lambda n^2}{4}=\frac{\lambda t(t-1)n^2}{8}$. Since $\rho_4(n^t,\lambda)\geqslant\rho(n^t,\lambda)$, $\rho(n^t,\lambda)=\rho_4(n^t,\lambda)=\lambda t(t-1)n^2/8$ by Theorem 3.1. □

Lemma 3.4. (Liang and Han, [3]) When *t* and λ are even, there exists a DRC-covering of λK_t with $\rho(t,\lambda)=\lambda t^2/8$ cycles.

Theorem 3.5. Let *n* be odd. When λ and *t* are even, $\rho(n^t,\lambda)=\lambda t[(t-1)n^2+1]/8$.

Proof. From [3] Theorem 3.8, there exists a DRC-decomposition of $2K_{n,n}$-$2K_2$ with $(n^2-1)/2$ cycles. Where $2K_2$ is a graph with *2*-repeat edge *{1,n+1}*. $2K_t(n)$-$\binom{t}{2}$($2K_{n,n}$-$2K_2$) form a graph $2K_t$, and the $2K_t$ can be covered by $t^2/4$ cycles from Lemma 3.4. Therefore, the $2K_t(n)$ can be covered by $\binom{t}{2}(n^2-1)/2+t^2/4=2[t(t-1)n^2+t]/8$ cycles. Let λ =*2k*. Each cycle in DRC-covering of $2K_t(n)$ repeats *k* times, we obtain the required DRC-covering of $\lambda K_t(n)$. □

4. Related results

In this section, we will discuss the case when the size of the cycles is uniform or is bounded.

Theorem 4.1. $\rho_3(n^t,\lambda)=\lceil tn/3\lceil\lambda n(t-1)/2\rceil\rceil+1$ when one of the following congruences is satisfied:

(1) $\lambda\equiv 2$ (mod *6*), $n\equiv 1$ or *2* (mod *3*) and $t\equiv 2$ (mod *3*);

(2) $\lambda\equiv 5$ (mod *6*), $n\equiv 2$ or *4* (mod *6*) and $t\equiv 2$ (mod *3*);

(3) $\lambda\equiv 5$ (mod *6*), $n\equiv 1$ or *5* (mod *6*) and $t\equiv 5$ (mod *6*),

and $\rho_3(n^t,\lambda)=\lceil tn/3\lceil\lambda n(t-1)/2\rceil\rceil$ otherwise.

Proof. From Theorem 4.1 in [7], we obtain the result. □

Theorem 4.2. When $n\equiv 0$ (mod *2*), there is $\rho_4(n^2,1)=n^2/4$. When $n\equiv 1$ (mod *4*), there is $\rho_4(n^2,1)=(n^2+n+2)/4$. When $n\equiv 3$ (mod *4*), there is $\rho_4(n^2,1)=(n^2+n)/4$.

Proof. When *n* is even, we obtain $\rho_4(n^2,1)=n^2/4$ from Theorem 3.3. When *n* is odd, from Lemma 2.2 in [8] and Proposition 2.1, we obtain the following table.

n (mod 4)	1	3
$\rho_4(n^2,1) \geqslant$	$(n^2+n+2)/4$	$(n^2+n)/4$

We distinguish three cases to show this theorem.

Case 1. When $n=3, 5, 7$

$K_{3,3}$ can be covered by 3 C_4's. They are

$(0,1,2,3,0), (0,1,4,5,0), (2,3,4,5,2).$

$K_{5,5}$ can be covered by 8 C_4's. They are

$(0,1,2,5,0), (2,3,4,7,2), (4,5,6,9,4), (1,6,7,8,1),$
$(0,3,8,9,0), (0,1,4,7,0), (2,3,6,9,2), (1,2,5,8,1).$

$K_{7,7}$ can be covered by 14 C_4's. They are

$(0,1,2,7,0), (2,3,4,9,2), (4,5,6,11,4), (6,7,8,13,6),$
$(1,8,9,10,1), (3,10,11,12,3), (0,5,12,13,0), (0,3,6,9,0),$
$(1,4,7,12,1), (2,5,8,11,2), (4,5,10,13,4), (1,6,9,12,1),$
$(2,3,8,13,2), (0,7,10,11,0).$

Case 2. When $n\equiv 1$ (mod 4) (by induction on n)

The theorem is true for $n=5$ as shown by Case 1. Suppose that the induction hypothesis is true for $K_{n,n}$. We will show that it is true for $K_{n+4,n+4}$. Let the vertices of $K_{n+4,n+4}$ be $0,1,...,n-1,A,B,C,D,n,n+1,...,2n-1,E,F,G,H$, and arrange them in this order. The C_4's of a DRC-covering of $K_{n+4,n+4}$ will be

- the $(n^2+n+2)/4$ C_4's of the covering of $K_{n,n}$,
- the $2n-2$ C_4's of a DRC-decomposition of $K_{n+3,n+3}$-$K_{n-1,n-1}$-$K_{4,4}$. The vertex set of $K_{n+3,n+3}$, $K_{n-1,n-1}$ and $K_{4,4}$. are $[0,n-2]\cup[n+1,2n-1]\cup\{A,B,C,D,E,F,G,H\}$, $[0,n-2]\cup[n+1,2n-1]$ and $\{A,B,C,D,E,F,G,H\}$, respectively,
- the 7 C_4's: $(A,B,n,E,A), (B,C,D,H,B), (A,D,n,G,A), (C,E,F,G,C),$
$(n-1,A,E,H,n-1), (n-1,C,D,F,n-1), (B,F,G,H,B)$.

One can check that all the edges of $K_{n+4,n+4}$ are covered and there are altogether $(n^2+n+2)/4+2n-2+7=[(n+4)^2+(n+4)+2]/4$ C_4 's. Therefore, $K_{n+4,n+4}$ satisfies the induction properties.

Case 3. When $n\equiv 3$ (mod 4) (by induction on n)

The theorem is true for $n=7$ as shown by Case 1. Suppose that the DRC-covering of $K_{n,n}$ has $(n^2+n)/4$ C_4's. The same as Case 2, we obtain that the DRC-covering of $K_{n+4,n+4}$ has $(n^2+n)/4+2n-2+7=[(n+4)^2+(n+4)]/4$ C_4's. □

Lemma 4.3. (see [4, 5]) Let m and n be positive integers with $m\geqslant 4$ even, and $n\geqslant 3$ odd. Then $C_m|K_{n,n}-I$ if and only if $m|n(n-1)$ and $m\leqslant 2n$.

Lemma 4.4. (Sotteau [6]) The bipartite graph $K_{r,s}$ can be decomposed into cycles of length $2k$ if and only if r and s are even, $r\geqslant k$, $s\geqslant k$, and $2k$ divides rs.

Theorem 4.5. Let m and n be positive integers with $m\geqslant 4$ even. When n is odd, $m\leqslant 2n$ and $m|n(n-1)$, there is $\rho_m(n^2,1)\geq n(n-1)/m+\lceil 2n/m\rceil$.

When n is even, $m\leqslant n$ and $m|n^2$, there is $\rho_m(n^2,1)\geq n^2/m$.

Proof. Since the degree of the nodes in $K_{n,n}$ is odd and the degree of the nodes of a cycle is even, there are at least *n* extra edges of difference in the covering. From Lemma 4.3 we have

$$\rho_m(n^2,1) \geq n(n-1)/m + \lceil 2n/m \rceil.$$

When *n* is even, from Lemma 4.4 we have $\rho_m(n^2,1) \geq n^2/m$. □

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