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Inverse Halftoning Based on Sparse Representation with Boosted Dictionary

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Abstract. Halftone image is widely used in printing and scanning equipment. It is significant for the halftone image to be preserved and processed. For the different resolution of the display devices, the processing and displaying of halftone image are faced with great challenges, such as Moore pattern and image blurring. The inverse halftone technique is required to remove the halftone screen. In this paper, we propose an inverse halftone algorithm based on sparse representation with the dictionary learned by two steps: deconvolution and sparse optimization in the transform domain to remove the noise. The main contributions of this paper include three aspects: first, we analysis the denoising effects for different training sets and the dictionary; Then we propose the denoising algorithm through adaptively learning the dictionary, which iteratively remove the noise of the training set and improve the dictionary; Then the inverse halftone algorithm is proposed. Finally, we verify that the noise level in the error diffusion linear model is fixed, and the noise level is only related to the diffusion operator. Experimental results show that the proposed algorithm has better PSNR and visual performance than state-of-the-art methods. The codes and constructed models are available at https://github.com/juneryoung2022/IH-WNNM.

Keywords: inverse halftoning, deconvolution, sparse representation, error diffusion.

1. Introduction

Halftone technology converts a continuous tone image into the binary version. Image halftoning can be seen as a map from R^N to $\{0,1\}^N$, where N is the dimension of image sizes. Image halftone algorithm is widely used in the scanning and printing process of newspaper, book, magazine and fax. Ordered dithering and error diffusion is the two main methods of halftoning [7,11], Fig. 1 (a) shows the halftone images that generated from the continuous tone color image with size of $256 \times 256 \times 3$ by using ordered dithering filter, which is the left part of Eq. (1). While Fig. 1 (b) and (c) are the halftone version by using error diffusion methods with Floyd and Jarvis halftone filters respectively. In the ordered dithering method, the halftone image is generated by comparing the pixels between the original image and the filter matrix correspondingly. The error diffusion technique compares the image pixel value with a fixed threshold value, and diffuses the quantization error into the neighbor pixels according to the weights. Floyd and Jarvis proposed the error diffusion algorithms with different filter generators [6, 9]. which are the middle and right parts of Eq. (1) respectively. Kite et al. proposed the linear model that simulate the error diffusion filter, and pointed out that error diffusion halftone image can be regarded as a convoluted continues tone image plus the noise. It is necessary to inverse the halftone image to continuous tone for the halftone version does not contain detail information of image and with bad visual defect. And the halftone image is not easy to be processed. For example, the halftone image should be inverse halftoned in the scanning process. In this paper, we proposed an effective inverse halftone algorithm, which reconstructs the continuous tone image form the halftone version.

(a) (b) (c)

Fig. 1. The ordered dithering halftone image (a), Floyd and Jarvis error diffusion halftone images (b) and (c) generated from the color *Lena* image with the filters in Eq. (1) respectively

$$(1/17) \times \begin{bmatrix} 8 & 11 & 7 & 10 \\ 15 & 1 & 64 & 4 \\ 6 & 9 & 5 & 12 \\ 13 & 3 & 14 & 2 \end{bmatrix}, \quad (1/16) \times \begin{bmatrix} 0 & \bullet & 7 \\ 3 & 5 & 1 \end{bmatrix}, \quad (1/48) \times \begin{bmatrix} 0 & 0 & \bullet & 7 & 5 \\ 3 & 5 & 7 & 5 & 3 \\ 1 & 3 & 5 & 3 & 1 \end{bmatrix} \qquad (1)$$

2. Related work

There are not effective inverse halftone methods and the noise removal algorithms are still the normally traditional approaches. Gaussian low-pass filtering inverse halftone [3] is a simple and fast method, but it cannot retain image edge and texture information effectively. The wavelet based methods reconstruct the inverse halftone image by transforming and processing the image in the wavelet domain [13, 22], in which the literature [22] divides the inverse halftone procedure into two steps: deconvolution and denoising. Literature [16] proposed the nonlocal regularization inverse halftone method, which contains two regularization procedure: the first one is total variation (TV) regularization based on the BM3D denoising algorithm; the second one is the post-processing by using the non-local regularization. The literature [25] descreens the image through training the descreening model, which contains two nonlinear operators: one is resolution synthesis-based denoising (RSD); and the other is SUSAN filtering. Literature [26] uses trained double dictionaries to reconstruct the continuous tone image. The two dictionaries are trained by the continuous image and the halftone version respectively, which using the same coefficient. The coefficient getting by the halftone image multiplies the dictionary trained using the continuous image, which can directly output the inverse halftone image. With the successful application of deep learning in image processing, the neural-network based methods have also been proposed and used in image inverse halftone reconstruction [8,10,15,17,24,27,28], which are in full using the prior information of inner images. Literature [34] address the inverse Halftone Colorization task, which tries to recover colorful images from black and white halftone prints, and can be treated as the joint problem of inverse halftone and colorization. In this literature the inverse halftone network architecture is extended from the state-of-the-art PRL model [32] where its content aggregation for synthesizing the global tone is replaced by a 7-layers U-Net to better support the analog halftoning. These methods directly train a map from halftone image to continuous tone image. And the map is used to inverse the halftone image. Sparse based methods [19,35], feature selection or fusing methods [36,37], Seeded random walk [30] and evolutionary algorithm [18] are machine learning based approaches, which show their well performance in classification and optimization problems. The approaches of machine learning and deep learning can directly inverse halftoning images without known the halftone filters. Although the results of these methods are well, but the training procedure is usually time consuming.

For the above problems, we need to find an effective algorithm to remove the halftone noise. Kite proposed the linear model of error diffusion and points out that the error diffusion halftone images can be seen as the combination of continuous tone image and add noises [11, 12]. Then the continuous tone image is obtained after denoising. There are three contributions in this paper listed as follows.

Firstly, we proposed an adaptive denoising algorithm based on sparse representation with trained dictionaries. By removing the noise of training image patches, the cleaning dictionaries can be output. In this way, both the cleaner dictionary and image can be got for several times training. Secondly, we proposed an inverse halftone algorithm. Similarly with the literature [22], we deconvolute the halftone image and output the continuous tone image with noise. Then we denoise the image by using the updated dictionaries in an iterative way. Thirdly, the gain in the linear error diffusion model proposed by Kite is a constant, which only relative with the operator of the linear model. We point out and verify experimentally that the noise level (denoted as σ) of the halftone image is also a constant.

The reasons for the better performance of our method can be explained that the clean image patches can get clean dictionary with good sparse representation ability compared with wavelet transformations [1,5]. And the output continuous image is the combination of the dictionary atoms. It is obviously that cleaner dictionary can be result in clean image. Here we use the patches of the denoised image to train the dictionary adaptively, where the patches size is 8×8 (the dimension size of the atom is 64). We improve the denoising performance for the image by removing the noise in the dictionary in each iteration.

3. Error diffusion linear model and deconvolution

3.1. Error diffusion model

Error diffusion halftone method is to quantify the gray image into halftone image with only two gray levels. The error diffusion model proposed by Floyd is shown as Fig. 2 (a), where $x(n_1, n_2)$ is the continuous tone image, (n_1, n_2) denotes the coordinates, $z(n_1, n_2)$ is the halftone image, $x'(n_1, n_2)$ is generated by diffusing the error $e(n_1, n_2)$ to the neighbor of $x(n_1, n_2)$ by the filter H. And H is the impulse response of $h(n_1, n_2)$. Different H can result in different error diffusion algorithms, such as Floyd and Jarvis error diffusion algorithm. The quantization error is diffused according to the weight in the neighbor field of the dot. The quantization model is nonlinear for using the quantizer to generate the halftone image. Kite et al. proposed the approximate linear model, which using a gain module K and additive white noise $\gamma(n_1, n_2)$ to simulate the effects of quantizer. The linear model is shown as Fig. 2 (b), using which the halftone image $z(n_1, n_2)$ can be denoted by the gray level image $x(n_1, n_2)$ and the additive white noise $\gamma(n_1, n_2)$.

$$z(n_1, n_2) = Px(n_1, n_2) + Q\gamma(n_1, n_2) = (p * x)(n_1, n_2) + (q * \gamma)(n_1, n_2) \quad (2)$$

where $*$ denotes the convolution operator, P and Q denote the linear time invariant (LTI) system.

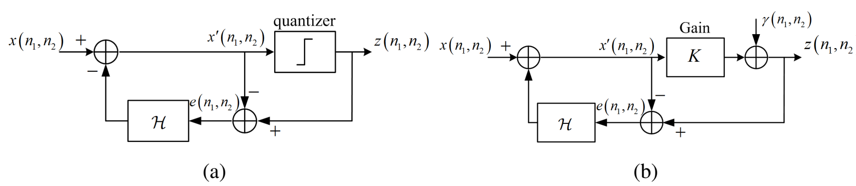


Fig. 2. Error diffusion model. The former is nonlinear error diffusion and the other is linear one

3.2. Halftone image deconvolution

The inverse halftone can be seen as a classic deconvolution problem. With the noise level $Q\gamma(n_1, n_2)$, The gray image $x(n_1, n_2)$ can be obtained from the deconvolution of the halftoning image $z(n_1, n_2)$ and the filter operator P. Kite et al. proposed the linear model of error diffusion.

$$Z(f_1, f_2) = \frac{KX(f_1, f_2) + (1 - H(f_1, f_2))R(f_1, f_2)}{1 + (K - 1)H(f_1, f_2)} \quad (3)$$

when

$$P(f_1, f_2) = \frac{K}{1 + (K - 1)H(f_1, f_2)}, \quad Q(f_1, f_2) = \frac{1 - H(f_1, f_2)}{1 + (K - 1)H(f_1, f_2)}$$

then

$$Z(f_1, f_2) = P(f_1, f_2)X(f_1, f_2) + Q(f_1, f_2)R(f_1, f_2) \tag{4}$$

from Eq. (4) we can get the following equation.

$$P^{-1}(f_1, f_2)Z(f_1, f_2) = X(f_1, f_2) + P^{-1}(f_1, f_2)Q(f_1, f_2)R(f_1, f_2) \tag{5}$$

Transforming Eq. (5) in Fourier domain, we can get the Eq. (6)

$$P^{-1}z(n_1, n_2) = x(n_1, n_2) + P^{-1}Q\gamma(n_1, n_2) \tag{6}$$

where the $P(f_1, f_2)$ and $Q(f_1, f_2)$ are the responses of P and Q in the frequency domain, which are the transformed functions of the signal and the noising respectively. $P(f_1, f_2)$, $Q(f_1, f_2)$, $H(f_1, f_2)$, $X(f_1, f_2)$, $Z(f_1, f_2)$ and $R(f_1, f_2)$ are the 2D Fourier transformation of $p(n_1, n_2)$, $q(n_1, n_2)$, $h(n_1, n_2)$, $x(n_1, n_2)$, $z(n_1, n_2)$ and $\gamma(n_1, n_2)$ respectively. Giving the error diffusion methods, such as the $h(n_1, n_2)$, K is constant for different images [12]. Such as $K \approx 2.03$ for the Floyd error diffusion method and $K \approx 4.45$ for Jarvis method. We will verify that the noise $\gamma(n_1, n_2)$ and the noise level are both constants for the error diffusion method. Inverse halftone can be regarded as the deconvolution of halftone image. The deconvolution algorithm of halftone image includes the following two steps.
Step 1: Convolution operation:
According to Eq. (2) and Eq. (3), the estimate noise image $y(n_1, n_2)$ of the input $x(n_1, n_2)$ can be got.

$$y(n_1, n_2) = P^{-1}z(n_1, n_2) = x(n_1, n_2) + P^{-1}Q\gamma(n_1, n_2) \tag{7}$$

Step 2: Denoising in the transform domain:
The Energy of the discontinuous edge of the image is distributed in many Fourier transform coefficients, while the deconvolution technology based on Fourier transform will result in ringing and blurring artifacts. This is the uneconomical representation of Fourier transform, where the coefficients are easy to confusion with noise. And the wavelet transform can provide a compact image edge sharpening representation. Therefore Neelamani et al. used the Wavelet transform to denoise [22]. The sparse representation based on the compact dictionary, which is more economical comparing the wavelet one [1,5]. We inverse the halftone image by using the sparse representation method based on the dictionary learning by using the K-SVD algorithm. The inverse halftone model can be shown in Fig. 3

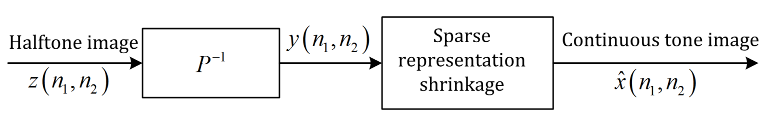


Fig. 3. Image inverse halftoning model

4. Image denoising based on sparse representation

4.1. Sparse and redundant representation of images

In recent years, the theory and application of sparse representation have been developed continuously [4, 23]. Sparse representation has achieved good results in such aspects as image restoration [21], denoising [5], super-resolution [33], face recognition [31] and so on. After transforming under the basis function, the signal can be expressed as a few non-zero coefficients, while most of the coefficients are zero or close to zero, then the signal is sparse or sparsely represented. The basic functions include Fourier transform (FFT), discrete cosine transform (DCT) and various wavelet transform. The method of optimal directions (MOD) has drawn renewed attention [2], which training a dictionary by using the signal vectors. The K-SVD algorithm [1] can train an completely dictionary adaptively and has a good sparse signal representation capability. Some methods such as sparse coding proposed by H. Lee [14], online dictionary learning proposed by Julien [20] and so on, can improve the calculation speed while maintaining well sparse capability.

4.2. Image patch denoising based on sparse representation

The literature [5] uses the image patches with size $\sqrt{n} \times \sqrt{n}$, $n = 8$, to construct a redundant dictionary with size $n \times k$, $k > n$. The sparse representation model show that each image patch x can be sparse represented

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_0 \quad s.t. \quad D\alpha \approx x \tag{8}$$

where the coefficient $\hat{\alpha}$ is sparse, that is $\|\alpha\|_0 \ll n$. The $\|\alpha\|_0$ denotes the number of non-zeros of α.

Assuming that x is sparse and polluted by additive zero-mean Gaussian noise of mean square error σ, the corresponding noisy image block y is obtained. For this image block, the following equation can be used to denoise

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_0 \quad s.t. \quad \|D\alpha - x\|_2^2 \leq T \tag{9}$$

where T is defined by ε and σ, and the denoised image is $\hat{x} = D\alpha$. Eq. (9) becomes the following form by changing the restriction to the penalty term

$$\hat{\alpha} = \arg\min_{\alpha} \|D\alpha - y\|_2^2 + \mu\|\alpha\|_0 \tag{10}$$

The above optimization problem is an NP hard problem. Matching and Baise Pursuit algorithms [2] can be very efficient in obtaining an approximate solution. Since Orthonormal Matching Pursuit (OMP) algorithm [29] is simple and effective, this paper chooses OMP algorithm to solve it.

4.3. Image denoising based on sparse representation

Based on the model in above section the image X with size of $\sqrt{N} \times \sqrt{N}$,$N \gg n$, can be denoised by dividing into $(\sqrt{N} - \sqrt{n} + 1) \times (\sqrt{N} - \sqrt{n} + 1)$ patches. The image

Algorithm 1 Image Denoising Based on Sparse Representation

Input: Giving image Y with addition Gauss white noise (the standard variance is σ).
Parameters: m is the number of iterations, and λ is the Lagrange operator.

1: Initialization: $X = Y$, $D =$ complete DCT dictionary.
2: Updating the following two phases alternately for m times.
   – *update the sparsity coefficient* $\{\alpha_{ij}\}$: suppose D and X are constant, resolve the following formulation by using OMP algorithm, get the sparse coefficients of each image patches.

$$\hat{\alpha}_{ij} = \arg\min_{\alpha} \|D\alpha - R_{ij}X\|_2^2 + \mu\|\alpha\|_0 \tag{12}$$

   – *update the dictionary* D: suppose X is constant, updating both the atom of the dictionary D and the corresponding coefficient $\{\alpha_{ij}\}$ by decomposing the singular values of the error matrix.
3: updating the image X: iteratively updating $\{\alpha_{ij}\}$ and D, the denoised image can be output by optimizing the formulation

$$\hat{X} = \arg\min_{X} \|X - Y\|_2^2 + \sum\nolimits_{ij} \|D\alpha_{ij} - R_{ij}X\|_2^2 \tag{13}$$

Eq. (13) is a simple quadratic term

$$\hat{X} = (\lambda I + \sum\nolimits_{ij} R_{ij}^T R_{ij})^{-1} (\lambda Y + \sum\nolimits_{ij} R_{ij}^T D\hat{\alpha}_{ij}) \tag{14}$$

Output: $\hat{X}$

denoising problem can be solved through optimizing the following equation with three penalty terms [5].

$$\{\hat{\alpha}_{ij}, \hat{X}\} = \arg\min_{\alpha_{ij}, x} \lambda\|X - Y\|_2^2 + \sum\nolimits_{ij} \mu_{ij}\|\alpha_{ij}\|_0 + \sum\nolimits_{ij} \|D\alpha_{ij} - R_{ij}X\|_2^2 \tag{11}$$

where the first term requests that the observing image Y is approximate equal with the denoised image X, and the second term requests that the number of non-zero coefficients, and μ_{ij} is the pre-defined weight of each image patch, and the last term request that each reconstructed patch $R_{ij}X$ can be denoted by the dictionary D and the coefficients α_{ij}, where the matrix R_{ij} denotes the picked patch in the (i, j) of the image. The denoised image can be output by optimizing Eq. (11). The dictionary D is important for the efficiency of the algorithm. The literature [5] proposed a denoising algorithm based on the sparse representation of image patches, which described as follow.

5. Image denoising and inverse halftoning

5.1. Image denoising based on adaptive sparse representation

We consider to improve the performance of the denoising algorithm by improving the dictionary quality. Based on the algorithm 1, an improved sparse presentation denoising algorithm is proposed. Adaptive dictionary learning refers to the use of noisy images patches as a training set to learn the dictionary [5]. The learned dictionary contains noise because the atoms of the initial dictionary are directly coming from the image training

Fig. 4. Nature image training set

set. We remove the noise in the training samples in a feedback way. The quality of the dictionary is gradually approaching to the ground truth training dictionary (GTD).

The sizes (or the redundancy) of the dictionary and the training set directly impact on the image denoising effect. For the image training set without noising, with the increasing of dictionary redundancy the denoising performance tends to be enhanced. It can be seen in Fig. 5 the two curves of denoising performance for GTD and global dictionary (GD) methods. For the image training set with noise, the bigger size of dictionary the more noise in the dictionary. When the redundancy of dictionary increases to a certain balance point, the performance of denoising tends to decrease. It can be seen in Fig. 5 the curves of denoising performance for adaptive dictionary (AD) method. In the improved denoising algorithm, the noise in the training set is removed iteratively in order to clean training set, which can approximate the ground truth (GT) image training set. At the same time, we increase the dictionary redundancy to enhance the denoising ability.

Using the noise image Y to training the dictionary, Abaron et al. proposed the K-SVD algorithm [1] to sparsely encode the training samples $\{y_i\}_{i=1}^N$ and update the atoms of the dictionary. The trained dictionary D can be generated by the following formula

$$\min_{D,\alpha}\{\|Y - D\alpha\|_F^2\} \quad s.t. \quad \forall i, \|\alpha_i\|_0 \leq T_0 \tag{15}$$

The training process is divided into two stages: sparse coding stage and dictionary atomic update stage. The proposed algorithm employs the noise image as the training set to train the dictionary, and then using the dictionary to denoise the image. The denoised image as the new version to training the new dictionary and so on.

$$\min_{\alpha}\{\|X - D^{(J)}\alpha\|_F^2\} \quad s.t. \quad \forall i, \|\alpha_i\|_0 \leq T_0 \tag{16}$$

where $\hat{X}^{(J)}$ and $D^{(J)}$ are the image and dictionary in the Jth training and denoising. The chart of the algorithm is shown in Fig. 6. Experiments results show that the quality of the dictionary is boosted and gradually stable in the first several iterations. The performance of image denoising is also boosted with the updated dictionary. The algorithm is shown as Algorithm 2.

5.2. Image inverse halftoning based on adaptive dictionary training

The proposed algorithm 2 is divided into two stages. The diagram is shown in Fig. 3, where Y and $\hat{X}^{(J)}$ are the same as $y(n_1, n_2)$ and $\hat{x}(n_1, n_2)$ respectively. Here Y is the

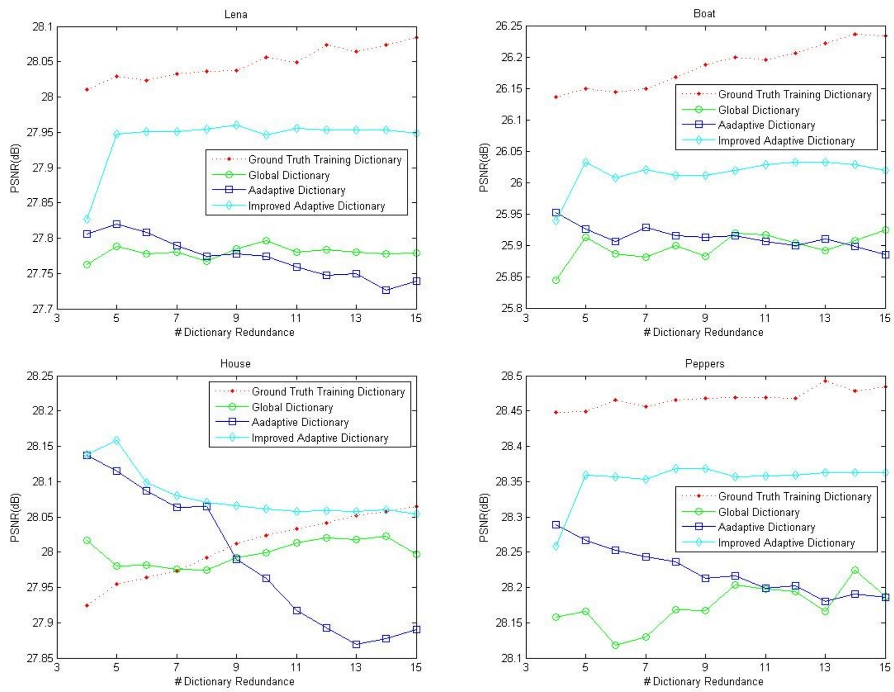


Fig. 5. The performance of denoising

noise image generated by deconvoluting the halftone image $z(n_1, n_2)$.

$$Y = P^{-1}z(n_,n_2) \tag{19}$$

Kite et al. have verified that the gain block K is constant for a certain error diffusion filter in the linear model. We declare that the noise level is constant and we verify the noise level with enough experiments. Given a noise image without known the noise level, we use the algorithm to remove the noise with different noise level parameters. We assume the noise level is the one that can result in the best PSNR value, which are shown in Fig. 7. We can see that the noise level is $\sigma \approx 42$. The same as Jarvis where the noise level is $\sigma \approx 18$. For P and Q are the linear filter operators, so the noise level of $\gamma(n_1, n_2)$ in the error diffusion model is constant.

When the noise level is fixed in the halftone image, the denoising algorithm can be explored to directly remove the noise generated in the inverse halftoning processing. It provides a way for setting noise levels when using other denoising algorithms for image inverse halftoning. It is reasonable to resolving image inverse halftoning by using the image denoising algorithm.

6. Experimental results and analysis

In this section we first demonstrate that the proposed algorithm can remove noise more effectively. For well comparing with the adaptive dictionary learning denoising algorithm

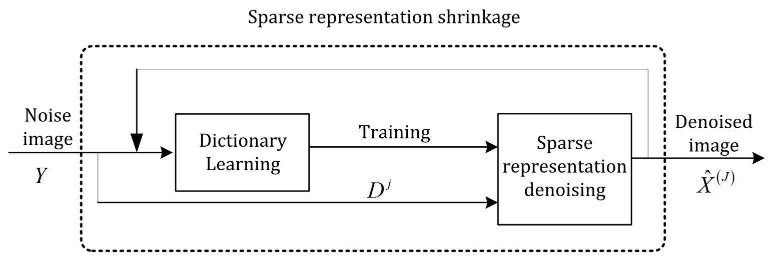


Fig. 6. Denoising by adaptive training dictionary iteratively

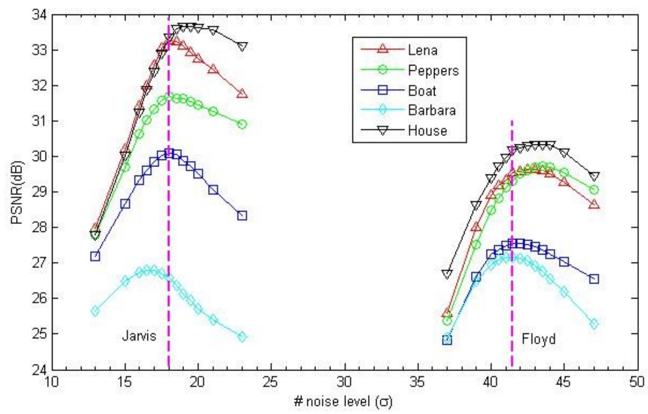
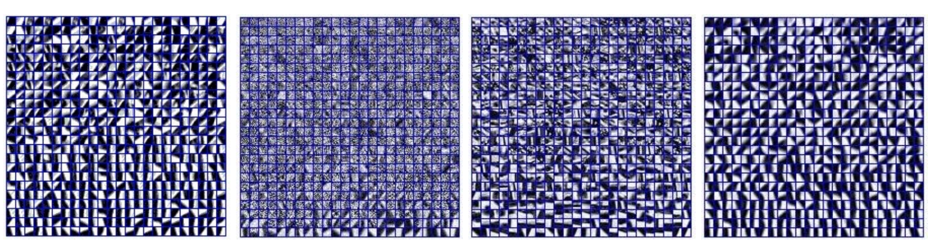


Fig. 7. The noise level of error diffusion model



(a) GT (b) AD (c) GTD (d) ID

Fig. 8. Different trained dictionaries

Algorithm 2 Image Denoising With Boosted Dictionary

Input: Giving image Y with addition Gauss white noise (the standard variance is σ).

Parameters: w is the number of iterations, J is the iteration number, and λ is the Lagrange operator.

1: Initialization: D^0 is the fixed number of samples randomly selected from the sample set after normalization. $Y^0 = Y$.

2: **for** $J = 1, \cdots, w$ **do**

3: Dividing the image $Y^{(J-1)}$ into patches as the training set $\{R_{ij}Y^{(J-1)}\}$. $D^{(J-1)}$ is formed by randomly selecting $(R_0 + J)$ sample. $D^{(J)}$ is got by training $D^{(J-1)}$. Set $X = Y^{(J-1)}$.

- *Sparse coding*: computing the sparse coefficients α_{ij} of the image patch in training set.

$$\hat{\alpha}_{ij} = \arg\min_{\alpha} \|D^{(J)}\alpha - R_{ij}X\|_2^2 + \mu\|\alpha\|_0 \tag{17}$$

- *Update the dictionary* D: suppose X is fix, in each time one of atoms in the dictionary and the corresponding correspond coefficient α_{ij} is updated.
- *Output* $D^{(J)}$

4: Sparse encoding: compute the coefficient α_{ij} of the noise image Y using Eq. (17).

5: Updated $D^{(J)}$ and the sparse encoding α_{ij} of noise image Y. The Jth denoised image can be computed.

$$\hat{X} = (\lambda I + \sum\nolimits_{ij} R_{ij}^T R_{ij})^{-1}(\lambda Y + \sum\nolimits_{ij} R_{ij}^T D^{(J)}\hat{\alpha}_{ij}) \tag{18}$$

6: $Y^{(J+1)} = \hat{X}$

7: **end for**

Output: $Y^{(J)}$

in the literature [5], zero-mean additive Gaussian white noise is used as the same as literature [5]. Then the proposed algorithm is applied to image inverse halftoning and compared with other benchmarks.

6.1. Experimental results of proposed method.

Literature [5] use the nature images without noise and the noise image itself to training the dictionaries respectively. The former one is named as global trained dictionary (GTD), and the other one is adaptive trained dictionary (AD). It as shown in Fig. 4, we select three different training images without noise to train the global dictionary. We name the dictionary trained by ground truth image as ground truth dictionary (GT), while the dictionary trained by our proposed method is called improved dictionary (ID). The dictionaries are shown in Fig. 8. It is obviously that the ID in our method is very similar with the GT objectively, while GTD is better than AD. At the same time the denoising performance of these dictionaries also show the same phenomenon subjectively, which is shown in Fig. 5 when the redundance is lower than 9. For convenience, we set the redundancy of all the dictionaries as 9, and the noise level is set as 50. All the algorithms are implemented in MATLAB 2015b on a 64bit Windows 10 Intel(R) Core (TM) i7 7500 CPU with 2.7 GHz and 8 GB RAM.

From the Fig. 8 we can see that the quality of ID is objectively better than the GTD and AD. The possible two reasons are declared as follows. In one hand the algorithm iteratively removes the noise of the image training set while the dictionary is the combination of the selected image patches in the initial training. Hence the dictionary quality will

Algorithm 3 Image Inverse halftoning Based on Sparse Representation With Boosted Dictionary

Input: Error diffusion halftone image $z(n_1, n_2)$.
Parameters: The error diffusion filter $h(n_1, n_2)$, Gain block K.

1: Step 1: Deconvolution
2: The estimated inverse halftone image with noise is obtained by using Fourier transform and inverse transform, which can be described as followed form using the Eq. (20).

$$Y = X + \nu \tag{20}$$

where the additive Gaussian white noise is $\nu = P^{-1}Q\gamma(n_1, n_2)$, and the ground truth image is $X = x(n_1, n_2)$, the inverse halftone image is $Y = y(n_1, n_2)$.
3: Step 2: Denoising based on sparse representation
4: Using the proposed algorithm 2 to adaptively training the dictionaries iteratively.

Output: $Y^{(J)}$ is the output inverse halftone image.

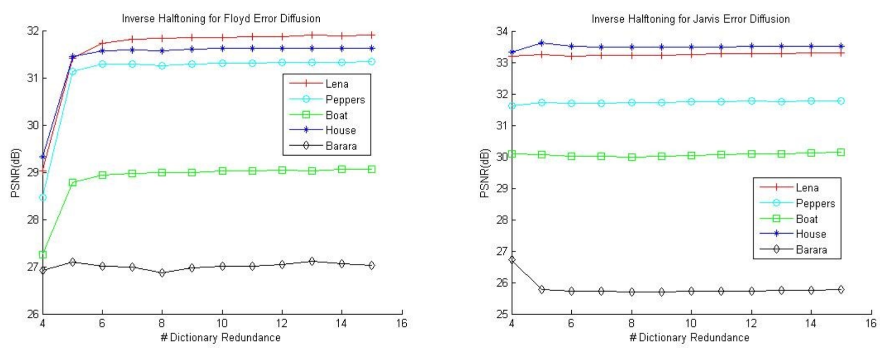
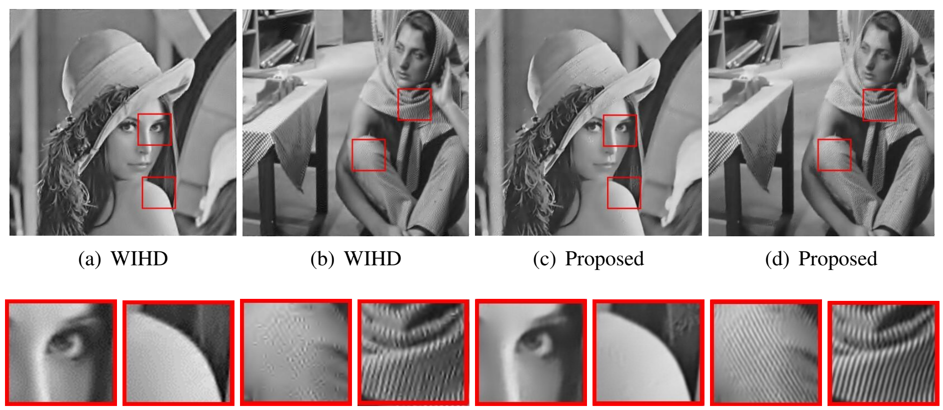


Fig. 9. The performance of proposed method

be boosted as the noise being removed iteratively. Then the performance of denoising is gradually improved. In the other hand, the redundancy of the dictionary will influence the denoising performance, so we increasing the redundancy in each iteration. Fig. 5 shows the denoising performance for several images with noise level $\sigma = 50$, which can be seen that the proposed algorithm can improve the denoising performance in the first several iterations and then tends to stable for *Lena*, *Peppers* and *Boat* images. For *House* image the performance of ID is better than the other three methods although the performance is not boosted with the iteration number increasing. Compared with the GTD, the proposed algorithm can improve the PSNR values of denoised images.

6.2. Experimental comparisons with benchmarks

Using the representative gray *Lena* and *Peppers* images with size 512×512, we compared the proposed algorithm with the benchmarks. We first halftoned the images with Floyd and Jarvis as the test halftone image, where we set the gain block K as 2.03 and 4.45 respectively. The deconvolution step is followed by the spare representation based denoising, and the PSNR results of the inverse halftone image are shown in Fig. 9 for the halftone images, where the noise level σ is set to 40 and 18 respectively. The objective results are shown in Fig. 10, where the proposed method performance well than the benchmark. The details of the inverse halftoned image are zoomed in 5 times. The subjective results are shown in Tab. 1, where the texts of bigger PSNR values are bold. We can see that the PSNR values of our proposed method are almost better than WIHD's for all the test images especially for Jarvis error diffusion halftone images.



(a) WIHD (b) WIHD (c) Proposed (d) Proposed

(e) (f) (g) (h) (i) (j) (k) (l)

Fig. 10. Comparing with the benchmark.(e)-(h) are the details of WIHD (zoomed in 5 times);(i)-(l) are the details of proposed ones

For color image, we also compared out method with the method of sparse representation with double dictionaries learning [26], where the image size is $512 \times 512 \times 3$. In our method the redundance of our dictionary is 8. We used the Floyd error diffusion halftone image as the same as Son et. al. In order to compare with literature [26] fairly, the redundancy of our dictionary is set as 8 and the noise level is set as $\sigma = 40$ in our method. It can be seen that the proposed method performance better than the benchmark proposed by Son et. al. subjectively in R, G and B channels, which is shown in Tab. 2. In Fig. 11, the visual performance of our method is better than the method in literature [26] objectively, especially for the whole bear body in the result image.

Table 1. The PSNR(dB) values of image inverse halftone methods.

Error Filter	Images	WIHD	Proposed	Error Filter	WIHD	Proposed
Floyd	*Lena*	31.96	**32.01**	Jarvis	32.81	**33.39**
	Barbara	31.04	**31.39**		31.16	**35.86**
	Boat	25.71	**27.03**		25.12	**25.79**
	Hill	31.24	**31.62**		31.82	**33.53**
	House	**29.19**	29.06		29.78	**30.15**

There are more artificial and blur textures in the output continuous tone image from the methods in literature [26] and [22], which can be seen in Fig. 10 and Fig. 11, while our proposed model can reserve the texture details and structure information.

(a) Ground Truth (b) Halftone image (c) Method in [26] (d) Our method

Fig. 11. The objective results for benchmarks

Table 2. The subjective PSNR(dB) values of benchmarks.

Dictionary	64×256			64×512			64×256		
Channel	R-ch	G-ch	B-ch	R-ch	G-ch	B-ch	R-ch	G-ch	B-ch
[26]	25.10	25.21	25.28	25.16	25.26	25.32	25.28	25.34	25.40
Proposed	**26.29**	**26.38**	**25.96**	**26.83**	**26.69**	**26.33**	**26.82**	**26.71**	**26.35**

7. Conclusion

In this paper, we first proposed an image denoising method by boosting a learning dictionary iteratively, which efficient for the deconvoluted halftone image. Then we proposed an image inverse halftone approach for error diffusion halftone image by using the deconvolution and the denoising algorithm. Furthermore, we verified that the noise level of the halftone image for error diffusion is the constant, which gives the rationality for the inverse halftoning algorithm with deconvolution and denoising successively. Researchers can combination all kinds of deconvolution and denoising models (including the popular deep learning networks) to explore inverse halftoning approaches. Experiment results show that our algorithm is superior to other benchmarks. The limitation of our method is that it only inverses the error diffusion halftone images effectively. We would find a versatile inverse halftone method for both the dot dithering and error diffusion halftone image. The similar work about image descreening is also a challenge work should be considered in future.

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