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Betweenness versus Linerank

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Abstract. In our paper we compare two centrality measures of networks, betweenness and Linerank. Betweenness is widely used, however, its computation is expensive for large networks. Calculating Linerank remains manageable even for graphs of billion nodes, it was offered as a substitute of betweenness in [12]. To the best of our knowledge the relationship between these measures has never been seriously examined. We calculate the Pearson's and Spearman's correlation coefficients for both node and edge variants of these measures. For edges the correlation tends to be rather low. Our tests with the Girvan-Newman algorithm [16] also underline that edge betweenness cannot be substituted with edge Linerank. The results for the node variants are more promising. The correlation coefficients are close to 1. Notwithstanding, the practical application in which the robustness of social and web graphs is examined node betweenness still outperforms node Linerank. We also clarify how Linerank should be computed on undirected graphs.

Keywords: big data, networks, centrality measures, betweenness, Linerank.

1. Introduction

As part of the ever more important big data analysis [19], the study of network centrality measures offers unique challenges [10, 18]. In a network centrality measures indicate the importance, interestingness of the nodes and the edges and they play a crucial role in many solutions to practical problems e.g. who are the most influential opinion-shapers in a community, which web pages contain the most relevant information about a certain topic [17] or which nodes should be deleted from a network in order to make the system to fall to pieces [3].

In our paper we compare different centrality measures, namely node and edge betweenness with node and edge Linerank respectively from different aspects. First, the Pearson's and Spearman's correlation coefficients are calculated both on real world and generated graphs. It turns out that the correlation between node Linerank and betweenness is higher than 0.9 almost in all cases, whereas for the edge versions it ranges from 0.2 to 0.7. These results suggest that node Linerank is a very promising candidate for substituting node betweenness, while this interchangeability is far more questionable for the edge variants. To further assess the applicability of Linerank we present the same correlation measures for approximates of betweenness, where instead of the $\mathcal{O}(nm)$ runtime of we perform a sampling in $\mathcal{O}(\sqrt{n}m)$ or as low as $\mathcal{O}(log(n)m)$ runtime.

After these initial results we study two practical applications of the betweenness measure and examine whether it can be substituted with Linerank without significantly worsening the performance of these methods. Firstly, we consider community detection using the Girvan-Newman algorithm. In our experiment instead of betweenness we calculated the Linerank value of the edges. In the comparison we used the same random benchmark graphs as in the calculation of the correlation coefficients. The results clearly show that the betweenness version significantly outperforms the Linerank version. On the one hand, this is not surprising since we have already observed that the correlation between these two measures is varying and it is never too strong. On the other hand, in their original paper Girvan and Newman tried three different variants of the betweenness measure and they found that the quality of the clusterings was not affected noticeably by the choice of the centrality measure. Our analysis reveals that this is no longer the case in the case of Linerank.

Secondly, we repeated the experiments of Boldi et al. in which they examined which nodes have the strongest impact in determining the structure of a network [3]. Or, in other words, which node-removal order influences this structure the most. They considered several centrality measures including Pagerank, harmonic centrality and betweenness. They removed the nodes in decreasing order according to these measures. Contrast to the Girvan-Newman algorithm however, in this case the order of the removal was fixed in the first step, which means that the aforementioned values were not recalculated after each deletion. The authors reported that in several cases betweenness outperformed the rest of the candidates. In our research instead of taking into account several centrality measures we focused solely on node betweenness and Linerank. Unlike in the previous case the difference between the performance of these two measures was unnoticeable for the generated benchmark graphs. However, in the case of real world graph networks betweenness outperformed Linerank again. This indicates that in practice one should still be careful when node betweenness is to be substituted with node Linerank.

The paper is organized as follows. In Section 2 the related work is presented. In Section 3 the algorithm used for approximating betweenness and its expected behaviour is described. In Section 4 the computation of Linerank is explained in more detail. Next, in Section 5 the results of our experiments are delineated. In Section 5.1 the Pearson's and Spearman's correlation coefficients are calculated. Then, in Section 5.3 edge betweenness is compared to edge Linerank by using the Girvan-Newman algorithm. Afterwards, in Section 5.4 the node variants are considered in order to determine the node removal order in networks and then to assess the influence of these removal orders. Finally, in Section 6 we conclude by summarizing our work. This paper is an extended version of the paper of the same name published at ICCCI 2014 [1].[1]

2. Related Work

In [12] centrality measures are divided into three families. The first group is constituted by the *degree related measures*, the second group consists of the *diameter related measures*, while the third group contains the *flow based measures*. We focus on the last group in our paper. Here, flow refers to the amount of information that may pass through a node or an edge. The most important member of this group, betweenness centrality, was proposed

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by Freeman[2]. For a given node v, it measures the ratio of those shortest paths that go through v. Formally, $v^{bet} = \sum_{u,w} \frac{b_{u,v,w}}{b_{u,w}}$, where $b_{u,w}$ and $b_{u,v,w}$ respectively denote the number of the shortest paths between nodes u, w and the number of those shortest paths from the previous ones that pass through v. The definition of this measure on edges can be formulated in a similar way.

Unfortunately, the computation of the exact values of betweenness is prohibitively expensive for large networks. For the 'node-variant' the best known algorithms work in time $\mathcal{O}(nm)$, where n denotes the number of nodes, while m the number of edges in a graph [12]. For this reason several attempts have been made to estimate the value of betweenness by using a carefully selected sample. As an orthogonal direction in [12] a new flow based centrality measure, Linerank, was introduced whose computation remains practically manageable even for graphs of billion nodes. As its name suggests the definition of Linerank was greatly inspired by Pagerank [17]. Roughly speaking, in the first step the original graph is transformed into the corresponding *line graph* on which the Pagerank values of the nodes are calculated. Since in a line graph the nodes represent the edges of the original graph by accomplishing the previous step one gains values measuring the importance of edges in a similar way as Pagerank measures the importance of nodes. However, we want to emphasize that in [12] this measure on edges has not been introduced, Linerank has been only defined on nodes. Our results below confirm that this was a wise decision indeed in the sense of the use case of substituting betweenness with Linerank. Nevertheless, in what follows we will refer to this measure as *edge Linerank*. In order to obtain a measure on nodes the previous scores of the incident edges of a node should be aggregated. The details will be given in Section 4.

Of the many approaches that exist for community detection, such as leader-driven community detection [20, 11] or mixed graph models [13], the Girvan-Newman community detection algorithm [16] is one of the most well-known. Here, edges are removed from the graph according to the decreasing order of their betweenness values. However, after the removal of the edge with the highest betweenness score the betweenness values of the remaining edges should be recalculated in each step. Sooner or later the graph falls to pieces and the resulting components are to be considered as communities. Of course, later these clusters may also be broken into to pieces. The hierarchy of communities is depicted by means of a dendrogram. Each level of this tree represents a possible clustering. In the last step the one with the highest modularity is chosen to be the final solution. In order to evaluate the performances of the betweenness and Linerank versions of the Girvan-Newman algorithm we applied *normalized mutual information*, since it is a widely used measure for testing the effectiveness of network clustering algorithms [8].

To generate random graphs, the model in [14] was used. This model generates graphs with communities, whose sizes vary according to a power law distribution with exponent β. The degree distribution is also assumed to be power law with exponent γ. Beside these parameters one can specify a mixing parameter μ s.t. each node shares a fraction $1-\mu$ of its edges with the nodes of its cluster and a fraction μ with the other nodes of the graph. The number of nodes is also given as a parameter.

[2] Strictly speaking, Anthonisse introduced this measure earlier than Freeman in a technical report, however, this work has never been published [16].

3. The Algorithm of Estimating Betweenness

The algorithm, which we have used in our comparisons [5], approximate the exact betweenness values by using a sample of size $\sqrt{n}$ or as low as $log(n)$, where n denotes the number of nodes in the graph. In the paper, where the state of the art method of computing the betweenness values is presented [4], the formula of the betweenness value of node v is rewritten in the following way:

$$v^{bet} = \sum_{u,w} \frac{b_{u,v,w}}{b_{u,w}} = \sum_{u,w} \delta(u,w,v) = \sum_{u} \delta(u,v), \text{ where } \sum_{w} \delta(u,w,v) = \delta(u,v).$$

Here, $\delta(u,v)$ is called the *one-sided dependency of u on v*. Basically, in [4] these one-sided dependencies are calculated for each node u by using a breadth-first search to find the shortest paths from u and then applying a cunning bottom-up labelling strategy, which results the desired betweenness values. In the estimation of [5] only a subset of the nodes are selected to calculate the one-sided dependencies. The theoretical justification of the method is provided by a result of Hoeffding [9], who has proven that for independent, identically distributed random variables $X_1, \ldots, X_k$ with $0 \leq X_i \leq M$ $(0 \leq i \leq k)$ and an arbitrary $\xi \geq 0$:

$$P\left(\left|\frac{X_1 + \ldots + X_k}{k} - E\left(\frac{X_1 + \ldots + X_k}{k}\right)\right| \geq \xi\right) \leq e^{-2k(\frac{\xi}{M})^2}.$$

In our case, for a randomly selected node p_i

$$X_i(v) = \frac{n}{n-1}\delta(p_i, v)$$

is used as a single estimate. Setting

$$M = \frac{n}{n-1}(n-2),\ \xi = \varepsilon(n-2)$$

Hoeffding's bound can be applied. Namely, since the expectation of estimate $\frac{1}{k}(X_1(v) + \ldots + X_k(v))$ is equal to the betweenness value of v Hoeffding's bound guarantees that the error of the approximation is bounded from above by $\varepsilon(n-2)$ with probability at least $e^{-2k\left(\frac{\varepsilon(n-2)}{\frac{n}{n-1}(n-2)}\right)^2}$, which is $e^{-2k(\frac{\varepsilon(n-1)}{n})^2}$ [5].

In [5] several strategies for random selection had been compared, and it turned out that in overall the method of selecting the nodes based on the uniform distribution outperforms the rest of the strategies. Thus, in our paper we also implemented this version. In [7] it is stated that $k \in \mathcal{O}(log(n))$ samples are sufficient for approximating closeness centrality. To benchmark our results with Linerank we measured the performance of betweenness and edge betweenness with both $\mathcal{O}(log(n))$ and $\mathcal{O}(\sqrt{n})$ samples.

4. The Computation of Linerank

As it has already been outlined in the introduction for a graph G Linerank is calculated by first constructing the line graph of G denoted by $L(G)$. In a line graph each edge of the

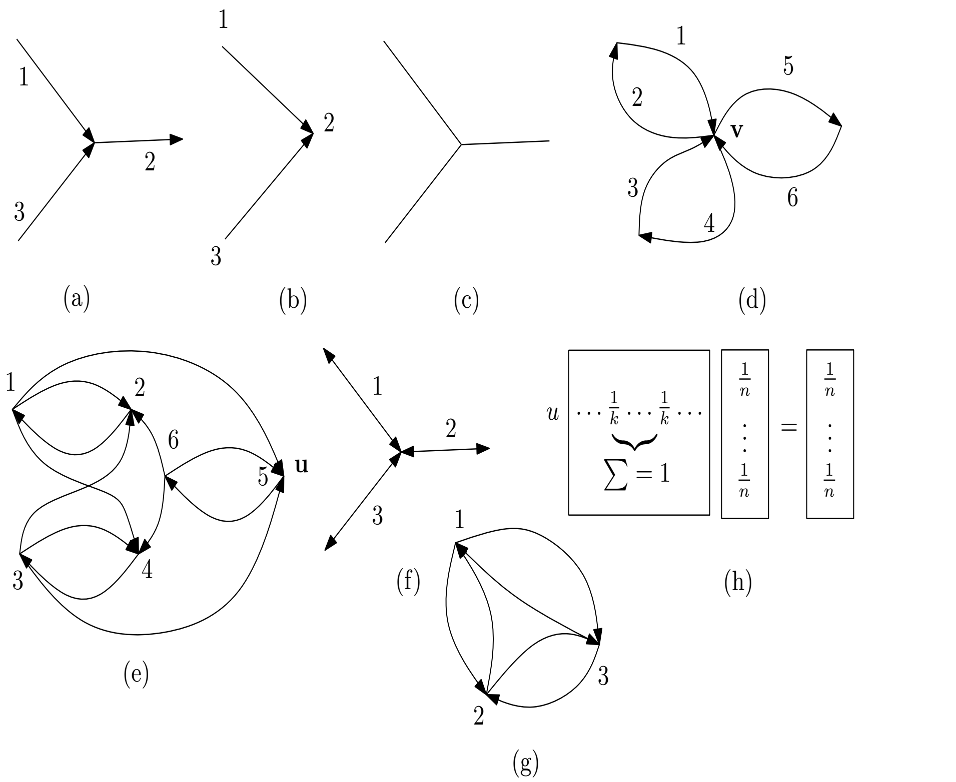


Fig. 1. (a) A graph G with weights. (b) $L(G)$. (c) An undirected graph H. (d) $\tilde{H}$. (e) $L(\tilde{H})$. (f) The result of substituting the undirected edges of H with bidirected edges. (g) The line graph of the graph on (f). (h) The matrix used in the computation of Pagerank values (proof of Corollary 1).

original graph is represented by a node. Let G be a directed graph and let $e_1 = (u_1, v_1)$, $e_2 = (u_2, v_2)$ be edges of G. In $L(G)$ there is an edge from the node representing e_1 to the node representing e_2, if and only if v_1 coincides with u_2, i.e., the target node of e_1 is the same as the source node of e_2. An example can be seen in Fig. 1. (a) and (b).

On the line graph a random walker at the current step either moves to a neighbouring node with probability β, or jumps to a random node with probability $1 - \beta$. If the walker moves to a neighbouring node, then she decides among the candidates according to the weights of the joining edges. We seek the stationary probabilities of this random walk. Or, to put in other words Pagerank is to be computed on the line graph. However, the size of the line graph can be much larger than that of the original graph which may render the explicit construction of the adjacency matrix unfeasible. Therefore, in [12] this adjacency matrix is decomposed into two sparse matrices by means of which the stationary probabilities can be computed efficiently. In the last step for each node of the original graph the scores of its incident edges are aggregated.

The original paper does not detail how Linerank should be calculated over undirected graphs. It is tempting to substitute each undirected edge with two oppositely directed edges. However, this approach would result in completely useless Linerank values. To be specific for graph G denote $\tilde{G}$ the result of the previous construction. Then the following statement can be proven.

Proposition 1 *Let G be an undirected graph. For each node u of $L(\tilde{G})$ the outdegree of each of the in-neighbours of u is the same as the indegree of u. (An in-neighbour is defined to be the source node of an ingoing edge of u.)*

Proof. First, note that if the indegree of a node v in $\tilde{G}$ is k, then the outdegree of v is also k, which is a straightforward consequence of the definition of $\tilde{G}$. The statement obviously follows from this observation, since in this case the indegree of the representative of an outgoing edge – denote it u – of v in $L(\tilde{G})$ is also k. What is more, the outdegree of each of the in-neighbours of u is also k as they correspond to the ingoing edges of v. Consider an example in Fig. 1. (c)-(e).

Corollary 1 *For an arbitrary undirected, uniformly weighted graph G the stationary probabilities – Pagerank values – are the same for each node of $L(\tilde{G})$.*

Proof. (Sketch.) Consider the matrix used in the computation of the Pagerank values of the nodes of $L(\tilde{G})$. From Proposition 1 it follows that the values of a tuple of this matrix are either 0's or $\frac{1}{k}$'s, where k is the indegree of the represented node by this tuple. What is more the sum of these $\frac{1}{k}$ values is equal to 1. Hence, when the product of this matrix and vector $(\frac{1}{n} \dots \frac{1}{n})$ is calculated at the first step of the computation, then the result is equal to the same $(\frac{1}{n} \dots \frac{1}{n})$ vector (Fig. 1. (h)), thus the computation terminates resulting the same Pagerank values (stationary probabilites) for all nodes.

Thus, instead of adding extra edges the line graph should be constructed as if the original undirected edges were bidirected. An example can be found in Fig. 1. (c), (f)-(g). Our experiments have shown that this construction avoids the preceding anomaly. What is more, it also saves a considerable amount of memory space.

5. Experiments

5.1. The Correlation between Linerank and Betweenness

As a first step in our investigation we calculated the betweenness and Linerank values of the nodes and the edges of real world graphs. In all cases the scatter plots suggested a strong correlation between the node Linerank and betweenness values, whereas for the edge variants the relationship remained somewhat blurred. As a typical example in Figure 2 (a) and (b) we have included the plots belonging to the polblogs dataset [2], a directed network of hyperlinks between weblogs on US politics recorded in 2005 with 1490 nodes and 19090 edges. Beside the aforementioned real world graphs we also conducted the same experiment on random graphs described in the introduction. Owing to the costly computation of the exact betweenness values we used graphs of rather smaller sizes, namely with 1000 and 5000 nodes. The γ exponent of the power law distribution of the degrees was set to 2, while the β exponent of the power law distribution of the sizes of the clusters was chosen to be 1. We further distinguished two cases. In the first case we worked with rather large clusters whose size ranged between 20 and 100 nodes, while in the second case this size ranged between 10 and 50. The mixing parameter varied between 0.1 and 0.6 with steps of 0.1. The plots revealed the same connection between the Linerank and betweenness values as in the case of the real world graphs.

Next, to quantify this relationship we calculated the Pearson's and Spearman's correlation coefficients of the two measures. For the polblogs dataset these values were high for the node variants of the centrality measures: Pearson's: 0.82 Spearman's: 0.89; while for the edge variants the correlation turned out to be much weaker: Pearson's: 0.15 Spearman's: 0.26. In the case of the random benchmark graphs we generated 10 graphs for every parameter settings and took the average of the results. In Figure 2 (c) and (d) the relevant diagrams can be found for graphs with 5000 nodes. The curves for the graphs with 1000 nodes look like almost exactly the same.

Interestingly, both for the node and edge variants the correlation between the centrality measures increases as the boundaries among the clusters becomes blurred. However, for the node versions it is very high in all cases and as the value of the mixing parameter grows the correlation approaches to 1, whereas for the edge variants the correlation becomes higher only when the clusters literally disappear from the graph.

5.2. Correlation between Betweenness and its approximations

Another possible route for providing feasible approximations of the algorithms described in Subsection 5.3 and 5.4 would be to approximate the measure of betweenness itself instead of substituting it as suggested in [5]. Compared to [5] we use different indicators to assess the results, a namely Spearman's and Pearson' correlation coefficients as in Section 5.1.

To demonstrate the efficiency of the approximation we have plotted the results for the polblogs dataset [2], that has also been used for the experiments in Subsection 5.1. The correlation coefficents were measured as presented in Table 1.

The node betweenness approximating algorithms performed exceptionally well from $\sqrt{n}$ samples and decently from $log(n)$ samples. The performance of Linerank shown in Subsection 5.1 is between these solution, however for large graphs Linerank has even

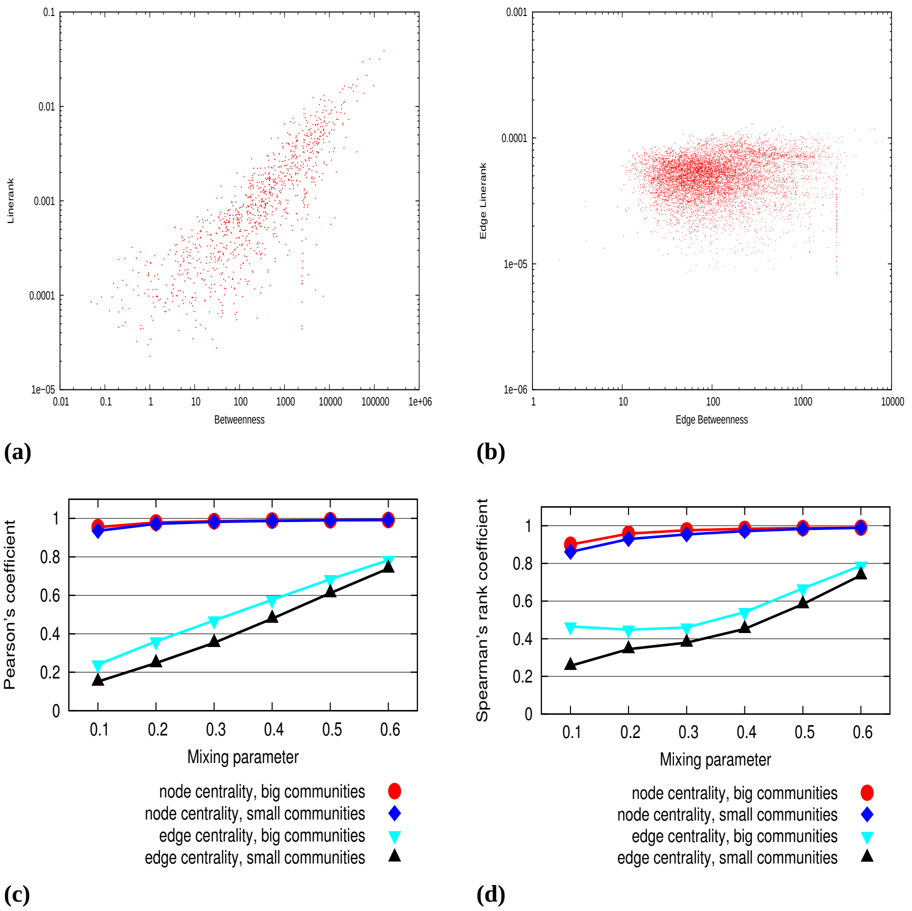
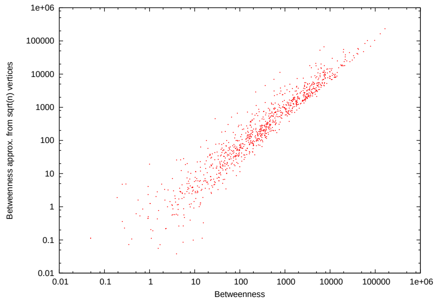


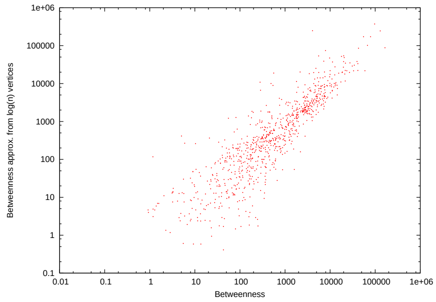
Fig. 2. (a) Node betweenness and Linerank values for the polblogs dataset. (b) Edge betweenness and Linerank for the same dataset. (c) Pearson's correlation coefficients for the benchmark graphs with 5000 nodes. (d) Spearman's correlation coefficients for the same set of benchmark graphs.

Table 1. Correlation for approximating betweenness values on the polblogs dataset

Coeff.	Node appr., $\sqrt{n}$	Node appr., $log(n)$	Edge appr., $\sqrt{n}$	Edge appr., $log(n)$
Pearson	0.87	0.75	0.34	0.16
Spearman	0.97	0.71	0.81	0.66

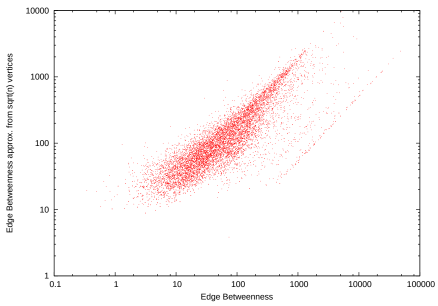


(a)

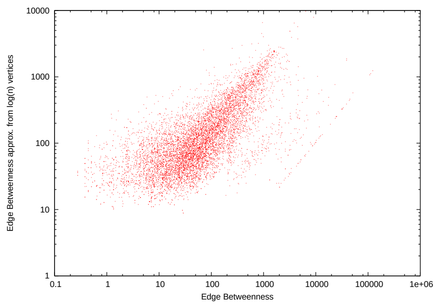


(b)

Fig. 3. (a) Node betweenness and Node betweenness approximation from $\sqrt{n}$ values for the polblogs dataset. (b) Node betweenness and Node betweenness approximation from $log(n)$ values for the for the same dataset.



(a)



(b)

Fig. 4. (a) Edge betweenness and Edge betweenness approximation from $\sqrt{n}$ values for the polblogs dataset. (b) Edge betweenness and Edge betweenness approximation from $log(n)$ values for the for the same dataset.

lower runtime than the one using $log(n)$ sample vertices. The edge variant of the measure is significantly less promising. The approximation itself is not on par with its node counterpart, but still outperforms edge Linerank in both cases.

We have also conducted experiments on the benchmark graph dataset described in Subsection 5.1. The baseline for the result were the results given by the exact betweenness computation, the results for the graphs with 5000 nodes summarized in Figure 5 also suggest that node Linerank can be a strong candidate in comparison with the approximate versions of node betweenness. Interestingly enough in certain cases edge Linerank outperformed the appoximations of edge betweenness, but the correlation tended to be rather low in almost all cases for the edges.

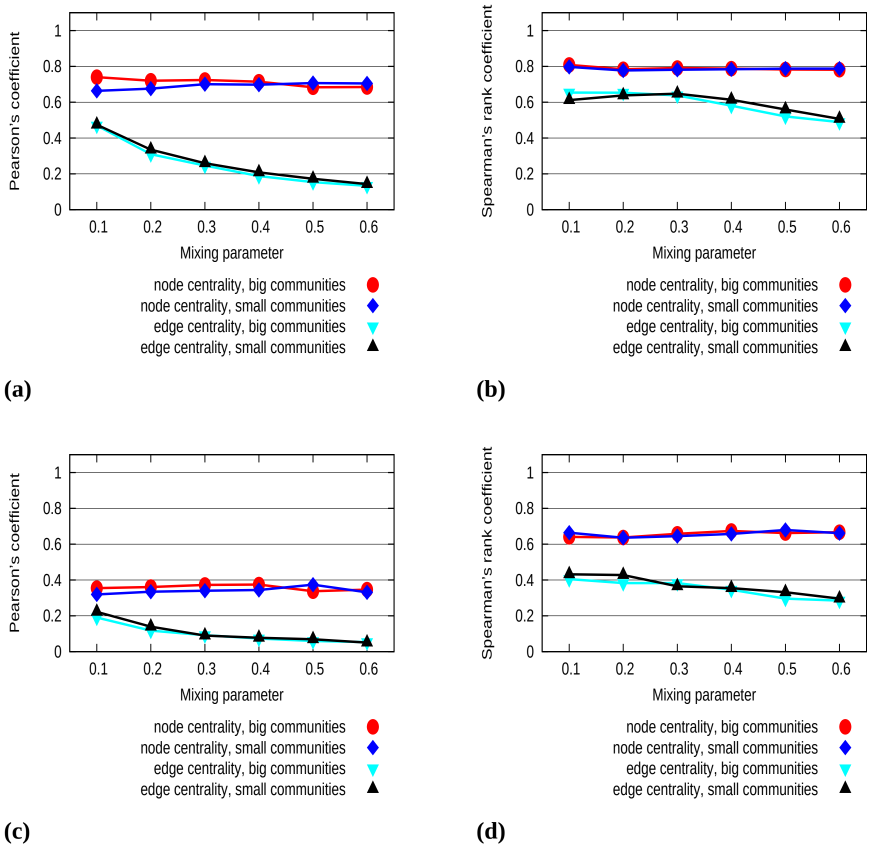


Fig. 5. (a) Pearson's correlation coefficients for the benchmark graphs with $\sqrt{n}$ samples. (b) Spearman's correlation coefficients for the same dataset with $\sqrt{n}$ samples. (c) Pearson's correlation coefficients for the same dataset with $log(n)$ samples. (d) Spearman's correlation coefficients for the same dataset with $log(n)$ samples.

5.3. Edge Betweenness versus Edge Linerank, the Girvan-Newman Algorithm

In order to assess the applicability of the edge Linerank in practice we implemented two versions of the well-known Girvan-Newman algorithm [16] for detecting communities in a network. The first version uses edge betweenness for finding the next edge to remove from the graph as in the original paper, whereas the second applies edge Linerank for this purpose. To compare the performance of the two variants we employed *normalized mutual information* [6], which is a frequently used measure for testing community detection algorithms.

For two partitions $\mathcal{X}, \mathcal{Y}$ define two random variables X and Y s.t.

$$P(X = i) = \frac{n_i^{\mathcal{X}}}{n} \text{ and } P(Y = j) = \frac{n_j^{\mathcal{Y}}}{n},$$

where $P(X = i), P(Y = j)$ denote the probability that a node belongs to the i^{th} and j^{th} cluster in partitions $\mathcal{X}$ and $\mathcal{Y}$ respectively, while $n_i^{\mathcal{X}}$, $n_j^{\mathcal{Y}}$ denote the number of nodes in these i^{th} and j^{th} clusters, finally n is the overall number of nodes. Accordingly, the joint distribution of these variables is defined as

$$P(X = i, Y = j) = \frac{n_{ij}}{n},$$

where n_{ij} denotes the number of nodes in the intersection of the aforementioned i^{th} and j^{th} clusters. The *mutual information* of two random variables is defined as

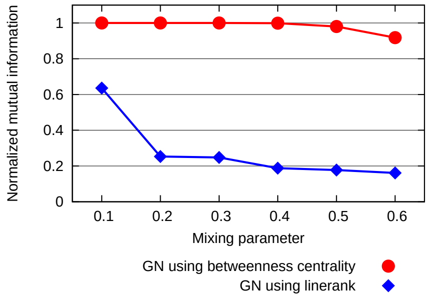
$$I(X, Y) = H(X) - H(X|Y),$$

where $H(Z)$ denotes the Shannon entropy of random variable Z. Thus, this measure tells how much the knowledge of Y reduces the uncertainty of X. As it is noted in [8] mutual information is not an ideal similarity measure, since for all subpartitions $\mathcal{Z}'$ of partition $\mathcal{Z}$ the mutual information of the derived random variables Z and Z' will be always the same, even though these subpartitions may substantially differ from each other. Therefore, in [6] *normalized mutual information* is introduced

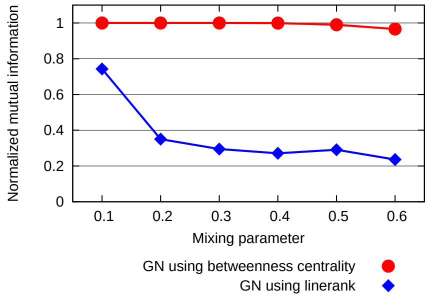
$$I_{norm}(X, Y) = \frac{2I(X, Y)}{H(X)H(Y)},$$

which equals 1, if the two partitions are the same, while for independent random variables its expected value is 0 [8]. This observation at least partially explains the popularity of this measure for comparing community detection algorithms.

In our experiments we used the same set of random benchmark graphs as in the previous case. Again, we generated 10 graphs with every parameter setting and we took the average of the normalized mutual information values. The behaviour on graphs with 1000 and 5000 nodes was indistinguishable, therefore we only included the diagrams related to the graphs with 5000 nodes. As the plots in Fig. 6. (a) and (b) clearly show the betweenness version of the Girvan-Newman algorithm significantly outperforms the Linerank version. Indeed, the scores of the latter are extremely low, which indicates the unusability of this method in practice. Of course, one may anticipate this result from the observations of the previous subsection, however, the former experiments only revealed



(a)



(b)

Fig. 6. (a) Effectiveness of GN algorithm versions on the benchmark graphs with larger clusters. (b) The same information for the benchmark graphs with smaller clusters.

that the correlation between the edge Linerank and betweenness values was rather low especially when the graphs contained quite definite clusters, but they did not foretell the superiority of betweenness. What is more, although these results suggest the inapplicability of edge Linerank for detecting clusters, since the correlation in the case of more scattered graphs was higher, the measure may still prove to be useful in certain scenarios, where the presence of clusters is not so remarkable.

5.4. Node Betweenness versus Node Linerank, the Robustness of Networks to Node Removal

After testing the applicability of edge Linerank we tried to find out to what degree node betweenness can be substituted with node Linerank in a practical application. For this purpose using node Linerank and betweenness we conducted the experiment of Boldi et al. again in which they tested to what extent the node removals can disrupt the structure of the web and social networks [3]. More precisely, in the course of node removal ϑm edges are deleted, where m denotes the number of edges and $0 \leq \vartheta \leq 1$. In the first step one defines an order among the nodes by using a measure and then considering the nodes in decreasing order starts to remove their incident edges. As soon as the number of the deleted edges becomes greater than or equal to ϑm, the process stops. The authors were interested in how the node removal orders based on different measures influence the fraction of reachable pairs as ϑ increases. They also wanted to assess the divergence between the distance distributions of the old and new graphs. They tried several different approaches to measure these changes and they have found that the *relative harmonic-diameter change* reflects the differences the best.

Relative harmonic-diameter is defined as

$$\frac{n(n-1)}{\sum_{u \neq v} \frac{1}{d(u,v)}},$$

where u, v are nodes, $d(u, v)$ denotes their distance, i.e., the length of the shortest path between them, and n again denotes the number of nodes in the graph. Unreachable pairs

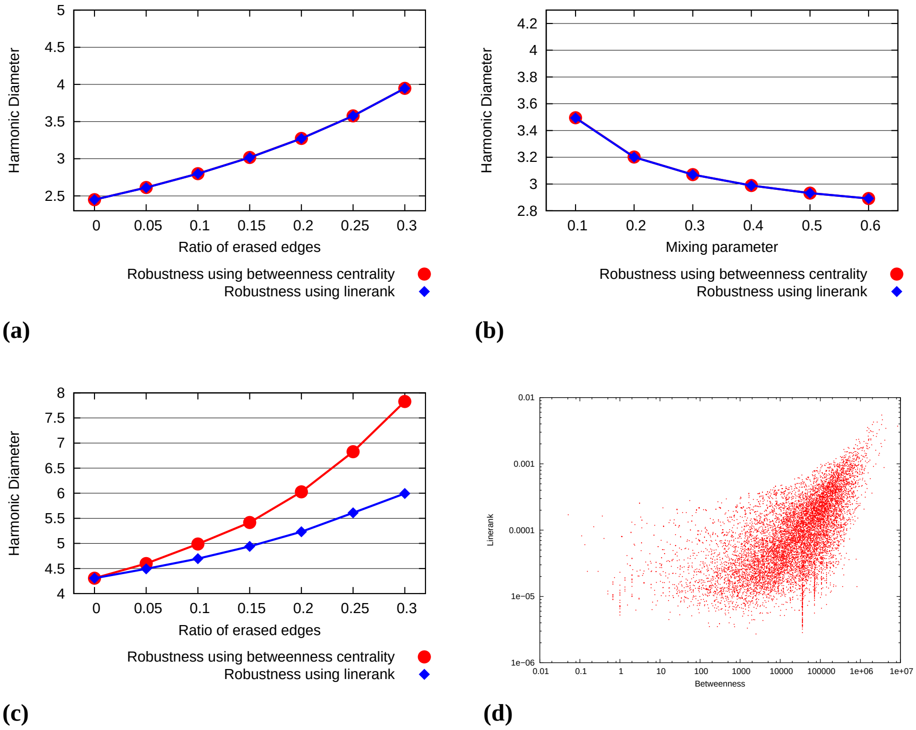


Fig. 7. (a) The relative harmonic-diameter change for the benchmark graphs with increasing ratio of the deleted edges (the mixing parameter is fixed). (b) The same data with varying mixing parameters (the ratio of the deleted edges is fixed). (c) The relative harmonic-diameter change for the CA-AstroPh network. (d) The scatter plot of the node betweenness and Linerank values for the CA-AstroPh network.

contribute 0 to the sum, hence this measure represents both disconnection and distance distribution [3]. For graph G denote $R(G)$ its relative harmonic-diameter, then the change in this measure is calculated as

$$\frac{R(Q)}{R(P)} - 1,$$

where P and Q respectively denotes the original graph and the graph after node removal.

In our own experiments again we used both generated and real world graphs. However, in this case we increased the number of nodes of the random graphs to 10000 and 50000. Accordingly, the sizes of the clusters also were also set higher. For the larger clusters these values ranged between 40 and 200, whereas for the smaller clusters between 20 and 100. The rest of the parameters remained the same. As the diagrams in Fig. 7. (a) and (b) show the results are indistinguishable for node Linerank and betweenness. We only plotted the data belonging to the benchmarks graphs with 50000 nodes and larger clusters, however, the rest of the diagrams look exactly the same. Neither the changes of the mixing parameter nor the increase in the ratio of deleted edges influences this behaviour.

Nonetheless, in the case of real world graphs the scenario is somewhat less straightforward. As one can see in Fig. 7. (c) for the CA-AstroPh dataset [15], which is the collaboration network from the e-print ArXiv in the Astro Physics category (nodes: 18772, edges: 198110), the difference between the relative harmonic-diameter change is more significant. On the other hand, as the scatter plot in Fig. 7. (d) suggests the correlation between node Linerank and betweenness is still high. We experienced the same phenomenon for several real world graphs, which indicates that although the correspondence between the two measures seems to be strong, in practice one should still be careful, when node betweenness is to be substituted with node Linerank.

6. Conclusions

In our paper we compared two flow based centrality measures betweenness and Linerank. We have found that in the case of edges the correlation between these measures varies but tends to be rather low. Our experiments with the Girvan-Newman algorithm also underlined that edge betweenness cannot be substituted with edge Linerank in practice. The results for the node variants are more promising. In our tests both Pearson's and Spearman's correlation coefficients were close to 1 in most of the cases. For the generated benchmark graphs this strong correspondence persisted in the practical application in which we examined the robustness of social and web graphs to node removal. However, for real world graphs, although the correlation seemingly remained high, node betweenness outperformed node Linerank. This which shows that even in this case the substitution of the former with the latter remains problematic. Beside these investigations we have also clarified how Linerank should be computed on undirected graphs.

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