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How MCDM Method and the Number of Comparisons Influence the Priority Vector

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Abstract. One of the most important issues in multi-criteria decision making is the number of requited judgments decision-maker/analyst has to perform. This paper presents a comparison of the results obtained by standard analytic hierarchy process (AHP), limited AHP, and best-worst method (BWM) if the number of criteria is 6, 7, and 8. The examples show that BWM's results are comparable with the results if standard AHP is used, while the limited version of AHP is generally inferior to the other two methods.

Keywords: analytic hierarchy process, best-worst method, criteria, comparison matrix size, number of judgments.

1. Introduction

Multi-criteria decision analysis (MCDA) is suitable for solving problems at different levels of complexity. Different sources and types of data (quantitative and qualitative; reliable and questionable; complete and incomplete, etc.) and the presence of conflicting elements (primarily criteria) influence the process of finding the final solution – decision. Multi-criteria decision-making methods (MCDMs) enable MCDA as a part of it and are broadly in use in diverse fields of science and practice. They combine knowledge and techniques of systems analysis, mathematics, computer science, social science, psychology, and many other disciplines in the complex world we live in. Most methods are considered heuristic because the rules they are based on are continuously subject to controversies among researchers. Namely, different methods may produce different results because of embedded different rules. Although the rules are usually clearly stated or intuitively respected there is not any proof that their application will lead to trustworthy output reflecting the judgment of decision-makers. MCDM methods perform differently in different environments such as individual and group context, availability or reliability of the information, quality and quantity of data, previous knowledge, expertise, and/or experience of the decision maker(s); and so forth.

As recently reported in [1] 'there are over a hundred MCDM methods, all aimed to simplify the problems and facilitate achieving optimal solutions'. The existence of a large number of methods may be seen as a strong point but also as a weakness because there is no absolute truth in any claim that a certain method is better than another for solving a given multi-criteria problem [2]. In several studies [3], [4], [5], [6], and [7]

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there are comparisons between different methods regarding problem structures, consistency, the accuracy of final results, ability to provide accurate representations of preferences of the decision-makers and the ability to comprehend the uncertainty, effects of aggregation and normalization method, etc. The studies [8] and [9] summarized the advantages, disadvantages, and areas of applications of numerous MCDM methods. Based on a review of psycho-cognitive literature and comparative study of different MCDA methods, in [2] are presented framework guidelines for selecting the method. [10] proposed a taxonomy for making the selection based on the problem type.

One of the most used general MCDM methods is the Analytic hierarchy process (AHP) developed by [11]. An analysis of collaboration evaluation in AHP research from 1982 to 2018 is presented in [12], where also some other surveys of the method and its wide applications can be found. Following the standard model, many different versions of AHP are developed in the next decades [13] but the axioms it follows have not been changed since it has been introduced in 1980. No significant changes in the methodology of its application happened meanwhile regarding hierarchization of the decision problem, preference measuring scales, manner of performing comparisons at all levels of hierarchy, and synthesis method to derive the final solution. Interpretation of output represented as cardinal information about priorities of decision elements at the bottom of the hierarchy (alternatives) versus heading element at the top of the hierarchy (goal) has not been changed, too. The determination of elements of the hierarchy is relatively objective and may depend on the degree of decision-making intervention (local, national or regional level). In most cases, the selected elements and factors of the decision matrix do not cover all possible criteria or all possible alternatives that may exist for only one goal. As it is reported in [14] 'one of the most prominent features of AHP methodology is to evaluate quantitative as well as qualitative criteria and alternatives on the same preference scale' so it is true to say that decision-makers need to be offered relatively simple methods to express their thoughts and preferences. Subjective decision makers' opinions can be interpreted by choosing a number from the numerical scale with predetermined appropriate semantic meaning [15].

Note that there are many modifications, versions, and optional uses of the AHP, which will not be discussed here, such as (1) Fuzzy AHP; (2) multiplicative AHP; (3) interval comparisons; (4) semi-empty comparison matrices; (5) individual versus group applications; or (6) hesitant AHP (AHP-H). The latest is proposed by [16] and it offers the decision-maker to give more than one opinion while making some judgments (pairwise comparisons). Instead of direct numerical expressions of judgments, linguistic preference relations are also in use. Discussion on the advantages and drawbacks of listed approaches, as well as on consistency measures and aggregation techniques in group decision-making frameworks, are out of scope in this work. Detail description of mentioned methods and techniques within 'the AHP hemisphere' can be found in rich literature and overviews in the subject area (e.g. [1], [17], [18], [19], [20], [21] [22], [23]).

Following the discussion on possible drawbacks in the AHP method, recall that the only significant modification of the original model has been the introduction of an ideal mode of synthesis to resolve the problem of re-ordering original alternatives if one original alternative is copied and added to the original alternative set. The other modifications in the base model, which will be labeled from now on as standard AHP

(AHP-S) does not exist. Rather, there are some additions in the methodology of AHP application, two of them being of interest here:

• Limited AHP (AHP-L), proposed by [24], for improving consistency of the decision-maker while using AHP-S; and

• Best-worst method (BWM), developed by Rezaei [25], [26] for reducing the number of multiplicative preference relations.

These two modifications tackle specific parts of the standard AHP method and do not change the structure or philosophy of the base method. They offer a relaxed environment to the decision-makers while they judge decision elements. Characteristics of AHP-L and BWM will be presented in the next section after the main characteristics of AHP-S are briefly described as preliminary information about the 'comparisons framework' created to analyze the application and output of all three methods for matrices of different sizes. Note that BWM is frequently treated as a different method from AHP, but in our opinion, it is only a very good modification of the judicial process and efficient addition to the original AHP methodology.

In this study, the authors firstly extracted three criteria sets consisting of six, seven, and eight criteria from three different AHP applications in water resources, agriculture, and environmental management. Weights of criteria computed in reported studies are copied here and used for comparisons with the results of the other two aforementioned methods.

The procedure labeled as Limited AHP (AHP-L) is performed by emptying three matrices and leaving in place only comparisons in the upper triangle next to the main diagonal. By strictly following the transition rule, remaining entries of matrices are filled with generated values and thus completely consistent matrices are created. Application of the eigenvector method on these matrices produced sets of criteria weights.

The third applied method is the best-worst method (BWM) which is methodologically different from AHP in part of optimizing weights for all sets of criteria by using a lower number of judgments than AHP-S.

The principal aim of the analysis of the results of the three methods is to demonstrate the sensitivity of solutions if different information (set of judgments) is available and to enable discussion of their opportunities in real-life multi-criteria decision making and AHP implementations.

The paper is organized in the following way. Section 2 provides a brief description and preliminary knowledge about the methods and approaches used in this study. Section 3 presents the results of the study. Conclusions are given in the final Section 4 followed by selected references.

2. Methods

2.1. Standard AHP (AHP-S)

The core of the original AHP [11], usually considered as the standard version of this method, lies in presenting the problem as a hierarchy and comparing the hierarchical elements in a pairwise manner by using Saaty's 9-point scale (Table 1) to express the importance of one element over another, in regards to the element in the higher level.

If n elements of one level of the hierarchy are compared regarding the element in the upper level, a comparison matrix (labeled also as multiplicative preference relation, MPR) has the following quadratic form:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}. \quad \textbf{(1)}$$

Each matrix element a_{ij} is a subjective judgment provided by the decision-maker of the mutual importance of the two elements, i and j. If the decision-maker is fully consistent, then the transitive rule $a_{ij}a_{jk} = a_{ik}$ should apply for all i, j, k in range 1 to n.

Table 1. Saaty's importance scale

Definition	Assigned value
Equally important	1
Weak importance	3
Strong importance	5
Demonstrated importance	7
Absolute importance	9
Intermediate values	2,4,6,8

Under perfect consistency, a_{ij} is equal to:

$$a_{ij} = w_i / w_j \quad \textbf{(2)}$$

where w_i and w_j are the local weights of elements i and j regarding the element in the upper level. So, the weight's vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$, which corresponds to the matrix (1), comprises the local weights of all the elements in the given hierarchy level regarding the element in the upper level.

However, vector $\boldsymbol{w}$ is unknown, and the problem is that there is no such unique vector because of the well-known inconsistencies of the decision-maker or the limitations imposed by Saaty's (or any other) scale. To measure the quality of the $\boldsymbol{w}$ vector, computed by any of the existing methods (e.g. [27], [1]), one can define several metrics and compare the original matrix A and corresponding matrix C:

$$C = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \dots & \dots & \dots & \dots \\ w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{bmatrix}. \tag{3}$$

A large number of elements in the hierarchy may not affect the level of congruence between matrices *A* and *C* due to the scale imperfections, insufficient knowledge about the decision problem of the decision-maker, etc.

The differences between corresponding elements of the matrices (1) and (3) are usually treated as inconsistencies of the decision-maker. In [28] and [11] is recommended the Consistency Ratio (*CR*) as a measure of individual inconsistency and it is considered as a part of the standard version of the method. The defined threshold value of 0.1 (less than this is considered as consistent) is arbitrary.

2.2. Limited AHP (AHP-L)

The standard analytic hierarchy process (AHP-S) evaluates decision criteria and alternatives by setting all multiplicative preference relations between decision elements at all levels of the hierarchy, then calculates local weights of criteria vs. goal, alternatives vs. each criterion, and finally synthesize local weights to calculate the final weights of alternatives vs. goal. The weights are plausible if the comparison matrices are consistent or near consistent [29], [30], [24], [31]. Consistency and ways to measure it are a subject of many controversies among researchers regarding their definition, interpretation, and usage. In standard AHP, full consistency is achieved only if the elements a_{ij} of given local comparison matrix *A* satisfy transitivity and reciprocal rule given as $a_{ij}=a_{ik}\cdot a_{kj}$ and $a_{ji}=a_{ij}^{-1}$ for all *i*, *j* and *k*, all ranging from 1 to *n*, where *n* is the size of a matrix *A*. For high-order matrices, say five and higher, the first rule is very difficult to reach and the inconsistency measurement is advisable. A good review of inconsistency measures is given in [32], while many aspects of their use can be found in rich literature (e.g. [33], [27], [34], [35], [36], [37]).

In [24] it is presented an expert module, implemented in Visual Prolog to assist the user in the construction of a consistent or near consistent matrix. The module is aimed to help the decision-maker to intervene after each comparison if inconsistency appears in his/her judgment. The user is guided to improve consistency through a sequence of four steps. The leading idea in this procedure is to use only *n*-1 judgments of decision-maker elicited at matrix diagonal adjacent to the principal diagonal where values ‘1’ are posted indicating all comparisons of decision elements with itself. Notice that the lower triangle of the matrix (as also in standard AHP) contains reciprocals of numbers in the upper triangle, symmetric to the main diagonal.

To illustrate the procedure of obtaining a fully consistent matrix of size 5, suppose that decision-maker compared by importance paired elements *C*1 with *C*2, *C*2 with *C*3, *C*3 with *C*4, and *C*4 with *C*5 (Figure 1). Values highlighted in green are from the Saaty’s scale (1/9, 1/8, ..., 1/2, 1, 2, ..., 8, 9) offered to be used by the decision-maker.

	C_1	C_2	C_3	C_4	C_5
C_1	1	1/2			
C_2		1	1/4		
C_3			1	3	
C_4				1	1
C_5					1

Fig. 1. Initial matrix required for the limited pairwise comparison method

By using numerical values (green highlighted numbers) from a diagonal next to principal diagonal, calculation of elements in the upper triangle of matrix A can be performed by applying the transitive rule as follows:

Row #1: $a_{13}=a_{12}\cdot a_{23}=(1/2)\cdot(1/4)=1/8$; $a_{14}= a_{12}\cdot a_{23}\cdot a_{34}=(1/2)\cdot(1/4)\cdot 3=3/8$;
$a_{15}=a_{12}\cdot a_{23}\cdot a_{34}\cdot a_{45}=(1/2)\cdot(1/4)\cdot 3\cdot 1= 3/8$

Row #2: $a_{24}=a_{23}\cdot a_{34}=(1/4)\cdot 3=3/4$; $a_{25}=a_{24}\cdot a_{45}=3/4\cdot 1=3/4$

Row #3: $a_{35}=a_{34}\cdot a_{45}=3\cdot 1=3$.

	C_1	C_2	C_3	C_4	C_5
C_1	1	1/2	1/8	3/8	3/8
C_2		1	1/4	3/4	¾
C_3			1	3	3
C_4				1	1
C_5					1

Fig. 2. Generated fully consistent matrix (AHP-L)

The resulting matrix in Fig. 2 (with reciprocals in its lower triangle, not shown) is fully consistent. Once calculated weights of elements C_1-C_5 by any of known prioritization methods (e.g. EV, AN, LLS, etc.), form priority vector of the matrix which can serve as a 'reference vector' to be targeted if any additional pairwise comparison is made by the decision-maker is available. That is, besides n-1 comparisons, up to (n-1)(n-2)/2 more comparisons must be added until the matrix is filled up. In the given example, to four 'green judgments' (n-1=5-1=4), additional 6 judgments have to add to the complete matrix as in standard AHP. The problem with this matrix is that although it is consistent, it is not real because it is not obtained by the decision-maker. Instead of n(n-1)/2=10 judgments, as in AHP-S, only half of it is real (made by the decision-maker) and the other half is generated. Besides that, a partly generated matrix is artificial, generated values may not belong to the scale used by the decision-maker; these values do not correspond to the semantics defined for values in the scale (see Table 1) and therefore are not justified. Finally, generated values may in some cases be out of scale.

2.3. Best-worst method (BWM)

Different from the basic idea of the AHP method where complete multiplicative preference relation is created by the decision-maker or analyst, the BWM executes the reduced number of comparisons of decision elements at a given level of the problem hierarchy (usually at criteria level). Multiplicative preference relation is incomplete because only requested is the preference of the best criterion over all the criteria and the preference of all criteria over the worst criterion.

Due to efficiency in reducing the number of required comparisons, the method received attention [38] and there are a quite number of reported studies on its use since the method has been originally proposed by Rezaei [25] as a non-linear model (4).

$$\begin{gathered}\min \max_{\mathrm{j}} \left\{\left|w_B/w_j - a_{Bj}\right|, \left|w_j/w_W - a_{jW}\right|\right\} \\ \text{s.t} \\ \sum_j w_j = 1, \quad \text{for all } j \\ w_j \ \geq 0 \quad \text{for all } j\end{gathered} \tag{4}$$

Following the ideas introduced in [33], [26] presented the linear model in which instead of minimizing the maximum value among the set of $\left\{\left|w_B/w_j - a_{Bj}\right|, \left|w_j/w_W - a_{jW}\right|\right\}$ minimization is performed over the maximums among the set of $\left\{\left|w_B - a_{Bj}\mathrm{w_B}\right|, \left|w_j - a_{jW}w_W\right|\right\}$. The problem is now:

$$\begin{gathered}\min \max_{\mathrm{j}} \left\{\left|w_B - a_{Bj}w_B\right|, \left|w_j - a_{jW}w_W\right|\right\} \\ \textbf{s.t.} \\ \sum_j w_j = 1, \quad \text{for all } j \\ w_j \ \geq 0 \quad \text{for all } j\end{gathered} \tag{5}$$

and corresponding linear version of the model (5) can be defined as the model (6).

$$\begin{gathered}\min \ \varepsilon \\ \textbf{s.t.} \\ \left|w_B - a_{Bj}w_{\mathrm{B}}\right| \leq \varepsilon, \quad \text{for all } j \\ \left|w_j - a_{jW}w_W\right| \leq \varepsilon, \quad \text{for all } j \\ \sum_j w_j = 1, \quad \text{for all } j \\ w_j \ \geq 0 \quad \text{for all } j\end{gathered} \tag{6}$$

In models (5) and (6) ‘for all *j*’ means ‘for all compared elements’ in the set of decision elements, either criteria, sub-criteria, or alternatives; for instance, if there are *n* criteria, then ‘for all *j*’ means *j* = 1, 2, ..., *n*.

Differently from the non-linear model (5) which may have multiple solutions, linear model (6) produces a unique solution: the optimal set of weights w_j^* for all j , and ε^*. Similar to Mikhailov’s model and his ‘natural measure of consistency’ μ^* [33] in the model (6) ε^*, with a value between 0 and 1, can be considered as an indicator of consistency demonstrated by the decision-maker. Obviously, the lower the value of this indicator, the higher the level of consistency.

So far, there is no clear suggestion of what would be tolerance limit regarding the consistency if the linear version of BWM is applied. Reported researches are focused on searching for the new consistency ratio to replace the original consistency ratio ε. One of the ongoing researches [39] is aimed at how to achieve a reflection of the consistency status of the input judgment instead of output results.

The number of multiplicative preference relations in BWM is lower than in AHP-S. In the case of BWM, there are n-2 Best-to-Others comparisons, plus n-2 Others-to-Worst comparisons, and one more Best-to-Worst comparison [25]. This gives a total of $2n$-3 comparisons, which is a lower value than $n(n-1)/2$ as in AHP-S. The difference between the numbers of required comparisons in the two methods rises with the size of the matrix. For instance, if the matrix size is 7, AHP-S requires 21 comparisons, whether this number in the case of BWM is only 11.

A review of the most recent studies in the field indicates an extension of the BWM method with other techniques such as those belonging to fuzzy sets theory. For instance, BWM is used to derive the priorities from hesitant fuzzy preferences in uncertain situations such as its integration in a fuzzy environment [40], by using hesitant numbers [41], [42], or grey numbers [43], [44].

3. Numerical Examples

3.1. Brief Introduction to the Examples

Several matrices are used to illustrate differences that appear if AHP-S, AHP-L, and BWM are used to determine the weights of decision elements. Matrices of sizes 6, 7, and 8 are taken from papers of the author team as published in peer-reviewed national and international journals. These matrices are created during real-life applications of the AHP-S method and then re-considered by the methods AHP-L and BWM. The following assumptions are adopted to unify conditions for comparing the results:

1. In all cases, Saaty's 9-point ration scale is used.
2. To apply the limited version of AHP, an upper triangle of each original matrix (from standard AHP application) is emptied except entries adjacent to the main diagonal. This way, only part of original judgments is left at the original place to enable generating remaining entries which will make such a new matrix fully consistent.
3. Prioritization method in AHP-S and AHP-L is performed by the eigenvector method.
4. BWM rating of remaining decision elements versus best and worst is performed following preferences obtained in the original AHP-S application.

Note that in real-life application decision-maker would be asked to fill diagonal in an empty matrix (if AHP-L is selected), to fill the upper triangle of the matrix (AHP-S), or to select the best and worst criterion and compare other criteria vs best and worst one (BWM). So, in each of the cases, the decision maker would express her/his judgments directly.

3.2. Example #1 – Matrix size 6

The source paper [45] presents an evaluation methodology based on five different prioritization methods within the AHP-S framework. The methodology is aimed at identification of the best among four alternative dispositions of pumping stations within the canal network of the Danube-Tisza-Danube system in northern Serbia. The selection process is based on six economic and technological criteria as follows: C_1 – economic parameters, C_2 – reliability of operation, C_3 – efficiency during flood events, C_4 – conformity with water supply operational schemes and rules, C_5 – adaptability to other infrastructural staging processes and C_6 – technical controllability.

The results obtained by three methods are as follows:

Standard AHP

The original matrix created by the decision-maker and weights of criteria derived by the *EV* method is presented in Table 2. The consistency ratio *CR* for this matrix is 0.089 which is less than 0.1 considered as the maximum permitted value to declare that the derived vector of weights is consistent and can be declared as a decision.

Table 2. AHP-S comparison matrix and weights of criteria (*Example #1*)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	Weight	Rank
C_1	**1**	1/5	1/3	1/3	1/5	1/5	0.042	6
C_2		**1**	3	2	3	3	0.341	1
C_3			**1**	2	1	1/2	0.146	4
C_4				**1**	1	3	0.170	2
C_5					**1**	2	0.164	3
C_6						**1**	0.136	5

Consistency measure: CR = 0.089 (<0.100; satisfactory)

Limited AHP

If all entries of the original matrix (Table 2) are deleted and only elements in the diagonal adjacent to the main diagonal are left (see green-highlighted numbers), then by application of the transition rule all 'emptied' entries are generated and presented in Table 3.

Table 3. AHP-L comparison matrix and derived weights of criteria (*Example #1*)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	Weight	Rank
C_1	**1**	1/5	3/5	6/5	6/5	12/5	0.103	3
C_2		**1**	3	6	6	12	0.513	1
C_3			**1**	2	2	4	0.171	2
C_4				**1**	1	2	0.085	4-5
C_5					**1**	2	0.085	4-5
C_6						**1**	0.043	6

A procedure for generating missing entries in the matrix of AHP-L was as follows:

Row #1: $a_{13}= a_{12}\cdot a_{23}$=1/5·3=3/5; $a_{14}= a_{13}\cdot a_{34}$=3/5·2=6/5; $a_{15}= a_{14}\cdot a_{45}$=6/5·1=6/5; $a_{16}=a_{15}\cdot a_{56}$=6/5·2=12/5

Row #2: $a_{24}=a_{23}\cdot a_{34}$=3·2=6; $a_{25}=a_{24}\cdot a_{45}$=6·1=6; $a_{26}=a_{25}\cdot a_{56}$=6·2=12

Row #3: $a_{35}=a_{34}\cdot a_{45}$=2·1=2; $a_{36}=a_{35}\cdot a_{56}$=2·2=4

Row #4: $a_{46}=a_{45}\cdot a_{56}$=1·2=2.

The partly generated matrix in Table 3 is fully consistent. However, it contains entries that do not belong to the 9-point scale and therefore do not have the exact linguistic (semantic) meaning given in Table 1. For instance, the generated value of element a_{16} is 12/5=2.4 which determines preferences among criteria *C*1 and *C*6 as between scale value 2 with semantic meaning '*C*1 is slightly more important than *C*6', and scale value 3 with semantic meaning '*C*1 is more important than *C*6'. It is difficult to differentiate preference between two criteria and to give a logical explanation for 'fine tuning' of 0.4 between 2 and 3.

The other point is that generated entry value may be outside the range of scale (1/9, 1/8, ..., 1, 2, ...9). For instance, generated entry a_{26} is 12, a value higher than 9 which stands for absolute preference of one element over the other. Note also that value 12 significantly differs from value 3 (*C*2 is strongly more important than *C*6), originally inserted by the decision-maker while using AHP-S.

Needless to say, is that in this case, the consistency ratio CR is zero as a consequence of the full implementation of transition rules.

BWM

If pairwise comparison vectors for the best criterion *C*2 and worst criterion *C*6 are defined as presented in Table 4 (by using as a 'direction of behavior' original preferences from AHP-S application), then the LP problem for this setting is:

$$
\begin{gathered}
\min\ \varepsilon \\
\textbf{s.t.} \\
|w_2 - 7w_1| \le \varepsilon \\
|w_2 - 6w_3| \le \varepsilon \\
|w_2 - 2w_4| \le \varepsilon \\
|w_2 - 4w_5| \le \varepsilon \\
|w_2 - 8w_6| \le \varepsilon \\
|w_1 - 2w_6| \le \varepsilon \\
|w_2 - 9w_6| \le \varepsilon \\
|w_3 - 5w_6| \le \varepsilon \\
|w_4 - 6w_6| \le \varepsilon \\
|w_5 - 4w_6| \le \varepsilon \\
w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1. \\
w_1, w_2, w_3, w_4, w_5, w_6 \ge 0
\end{gathered}
\tag{7}
$$

The solution to model (7) is as presented in the research paper [46] and reproduced in Table 5: $w_1 = 0{,}07402330$; $w_2 = 0{,}4126114$; $w_3 = 0{,}08636052$; $w_4 = 0{,}2590816$; $w_5 = 0{,}1295408$; $w_6 = 0{,}03838245$; $\varepsilon = 0{,}1055517$.

Table 4. BWM vectors for best and worst criterion (*Example #1*)

Criteria		C1	C2	C3	C4	C5	C6
Best criterion:	**C2**	7	1	6	2	4	8
Worst criterion:	**C6**	2	9	5	6	4	1

Table 5. BWM solution (*Example #1*)

Criteria	Weight	Rank
C1	0.074	5
C2	0.413	1
C3	0.086	4
C4	0.259	2
C5	0.129	3
C6	0.038	6

Value ε = 0.106 can be considered as satisfactory consistency of the process. Recall that the lower value (closer to zero) ε means better consistency.

Altogether (AHP-S, AHP-L, BWM)

Table 6 summarizes the results obtained by three methods.

Table 6. Summary of AHP-S, AHP-L, and BWM solutions (Example #1)

Criteria	Standard AHP (AHP-S)		Limited AHP (AHP-L)		Best-Worst Method (BWM)	
	Weight	Rank	Weight	Rank	Weight	Rank
C1	0.044	6	0.103	3	0.074	5
C2	0.340	1	0.513	1	0.413	1
C3	0.148	4	0.171	2	0.086	4
C4	0.166	2	0.085	4-5	0.259	2
C5	0.164	3	0.085	4-5	0.129	3
C6	0.139	5	0.043	6	0.038	6

Good agreement in the final ranking of criteria is between methods AHP-S and BWM, except on the two lowest positions. Recall that in the case of AHP-S there were $n(n\text{-}1)/2 = (6\cdot5)/2 = 15$ multiplicative preference relations, while in the case of BWM this number was $2n\text{-}3 = 2\cdot6\text{-}3 = 9$. The reduction of the number of comparisons by 40% (9/15) is significant and the result is not that much different, at least regarding the ranking of criteria at the first four positions. Weights of the top-ranked criterion C1 differ by 30% and for the second-ranked criterion C4 the difference is 56%; in both cases, higher values are obtained by the BWM. Weights for the third-ranked criterion C5 differ 27% and for the fourth-ranked criterion C3 weights differ by 82%; in the case of these two criteria higher values are obtained by the AHP-S. In AHP-L ranking of criteria is very different from the other two methods, except in the case of the top-ranked criterion which is also C2. It's very high weight is obviously due to high values of generated judgments a_{21}=5, a_{24}=6, a_{25}=6, and a_{26}=12, the last one even out of 9-point scale on its upper bound.

Remark #1.1. Applied three methods expectedly produced different weights of criteria which are following differences in methods. Although in all cases the same 9-point scale is used, the other prepositions are not followed in the same way. For instance, judgments in AHP-S and BWM are elicited from real decision-makers in two separate sessions, the second one (BWM) is undertaken several years after the first one (AHP-S) described in the source paper.

Remark #1.2. Because the evaluation of criteria is essential in any decision-making process, different weights obtained by different methods may significantly moderate the next steps of the evaluation process, such as the synthesis of local weights of alternatives by multiplication with the weights of criteria.

Remark #1.3. The number of real judgments in AHP-S is 15, in AHP-L only 5, and in BWM 9. The difference in information base used (number of real pairwise comparisons performed) for deriving weights is significant and richness of information is obviously on side of the first and third methods. In the case of AHP-L, it would be necessary to adjust the judgment of the decision-maker, for instance in an iterative way by the expert module proposed in [24]. However, this procedure we consider inappropriate, at least in the context we are analyzing here.

3.3. Example #2 - Matrix Size 7

In this source paper [47], a multi-criteria evaluation procedure is proposed for ranking five walnut cultivars selected in Serbia, Bulgaria, and France. Supported by AHP-S, the ranking process is settled with principal regard to nut characteristics of fruits. Eight criteria of different metrics are used for shaping and filtering two experts' preferences of criteria and cultivars. Here we elaborate the multiplicative preference relations contained in the matrix for criteria as obtained by one of the experts, a professor in fruit production and recognized national expert in walnut's and hazelnut's selection standards. The set of criteria was as follows: C1 – kernel's color, C2 – kernel's portion, C3 – nut's weight; C4 – the taste of the kernel, C5 – shell, C6 – storage, and C7 – trade value.

Standard AHP

Table 7. AHP-S comparison matrix and derived criteria weights (*Example #2*)

Criteria	C1	C2	C3	C4	C5	C6	C7	Weight	Rank
C1	**1**	3	1	7	5	9	7	0.321	1-2
C2		**1**	1/3	5	5	7	5	0.180	3
C3			**1**	7	5	9	7	0.321	1-2
C4				**1**	1	3	3	0.059	4
C5					**1**	3	1	0.053	5
C6						**1**	1/3	0.023	7
C7							**1**	0.042	6

Consistency measure: CR = 0.047 (<0.100; satisfactory)

The result obtained by the eigenvector prioritization method during standard AHP application is presented in Table 7. Consistency measure *CR* is at a very low value (CR=0.047), far below limit *CR*=0.01. Therefore, the vector of criteria weights can be considered as consistently derived.

Limited AHP

The results obtained by the eigenvector prioritization method during AHP-L application (with original judgments highlighted in green, and remaining judgments generated by the application of transition rule), are presented in Table 8.

Table 8. AHP-L comparison matrix and derived criteria weights (Example #2)

Criteria	C1	C2	C3	C4	C5	C6	C7	Weight	Rank
C1	**1**	3	1	7	7	21	7	0.356	1-2-3
C2		**1**	1/3	7/3	7/3	7	7/3	0.119	4
C3			**1**	7	7	21	7	0.356	1-2-3
C4				**1**	1	3	1	0.051	5-6
C5					**1**	3	1	0.051	5-6
C6						**1**	1/3	0.017	7
C7							**1**	0.356	1-2-3

Row #1: $a_{13}=a_{12}\cdot a_{23}=3\cdot 1/3=1$; $a_{14}=a_{13}\cdot a_{34}=1\cdot 7=7$; $a_{15}=a_{14}\cdot a_{45}=7\cdot 1=7$; $a_{16}=a_{15}\cdot a_{56}=7\cdot 3=21$; $a_{17}=a_{16}\cdot a_{67}=21\cdot 1/3=7$

Row #2: $a_{24}=a_{23}\cdot a_{34}=1/3\cdot 7=7/3$; $a_{25}=a_{24}\cdot a_{45}=7/3\cdot 1=7/3$; $a_{26}=a_{25}\cdot a_{56}=7/3\cdot 3=7$ $a_{27}=a_{26}\cdot a_{67}=7\cdot 1/3=7/3$

Row #3: $a_{35}=a_{34}\cdot a_{45}=7\cdot 1=7$; $a_{36}=a_{35}\cdot a_{56}=7\cdot 3=21$; $a_{37}=a_{36}\cdot a_{67}=21\cdot 1/3=7$

Row #4: $a_{46}=a_{45}\cdot a_{56}=1\cdot 3=3$; $a_{47}=a_{46}\cdot a_{67}=3\cdot 1/3=1$

Row #5: $a_{57}=a_{56}\cdot a_{67}=3\cdot 1/3=1$

Like in the other examples, consistency measure *CR*, in this case, is also zero due to the implementation of transition rules.

BWM

Pairwise comparison vectors are defined as in Table 9.

Table 9. BWM vectors for best and worst criterion (Example #2)

Criteria		C1	C2	C3	C4	C5	C6	C7
Best criterion*:	**C1**	1	3	1	7	5	9	7
Worst criterion:	**C6**	9	4	8	3	2	1	2

***Best** could also be **C3**

The corresponding optimization model is:

$$
\begin{gathered}
\min \varepsilon \\
\textbf{s.t.} \\
|w_1 - 3w_2| \le \varepsilon \\
|w_1 - 1w_3| \le \varepsilon \\
|w_1 - 7w_4| \le \varepsilon \\
|w_1 - 5w_5| \le \varepsilon \\
|w_1 - 9w_6| \le \varepsilon \\
|w_1 - 7w_7| \le \varepsilon \\
|w_2 - 4w_6| \le \varepsilon \\
|w_3 - 8w_6| \le \varepsilon \\
|w_4 - 3w_6| \le \varepsilon \\
|w_5 - 2w_6| \le \varepsilon \\
|w_7 - 2w_6| \le \varepsilon \\
w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 = 1 \\
w_1, w_2, w_3, w_4, w_5, w_6, w_7 \ge 0
\end{gathered}
\tag{8}
$$

The solution to this model is (Table 10): $w_1 = 0.303761$; $w_2 = 0.123754$; $w_3 = 0.371263$; $w_4 = 0.053038$; $w_5 = 0.074253$; $w_6 = 0.040180$; $w_7 = 0.033751$; $\varepsilon = 0.067502$. Value ε = 0.068 is close to zero and the solution can be considered as satisfactory like in Example #1.

Table 10. BWM solution (Example #2)

Criteria	Weight	Rank
C1	0.304	2
C2	0.124	3
C3	0.371	1
C4	0.053	5
C5	0.074	4
C6	0.040	6
C7	0.034	7

Altogether (AHP-S, AHP-L, BWM)

Table 11 summarizes the result obtained by the three methods.

All methods are in relatively good agreement regarding the final ranking of criteria, especially on the top three positions. The sum of weights of the top three criteria *C*1, *C*2, and *C*3 is 0.822 in AHP-S, 0.831 in AHP-L, and in BWM this sum is 0.799. The difference in these sums between methods of up to 4% is not significant. The sum of weights of the remaining four criteria in all cases is small and the final AHP synthesis across all criteria should result in approximately the same order of walnut cultivars if weights obtained by the AHP-L or the BW methods are used instead of weights obtained by AHP-S.

The results are interesting, especially from the standpoint of information available. In this example, AHP-S required $n(n-1)/2=(7\cdot6)/2=21$ comparisons, whether in BWM this number was $2n-3=2\cdot7-3=11$; in case of AHP-L, only 7 comparisons were required, taken from the original matrix (highlighted entries above the main diagonal in Table 8).

Reduction in the number of comparisons in AHP-L and BWM concerning AHP-S by 48% (11 vs. 21) and 67% (7 vs. 21) respectively, can be considered very significant while the result is not that much different regarding the ranking of decision elements.

Table 11. Summary of AHP-S, AHP-L, and BWM solutions (Example #2)

Criteria	**Standard AHP (AHP-S)**		**Limited AHP (AHP-L)**		**Best-Worst Method (BWM)**	
	Weight	Rank	Weight	Rank	Weight	Rank
C1	0.321	1-2	0.356	1-2	0.304	2
C2	0.180	3	0.119	3	0.124	3
C3	0.321	1-2	0.356	1-2	0.371	1
C4	0.059	4	0.051	4-5-6	0.053	5
C5	0.053	5	0.051	4-5-6	0.074	4
C6	0.023	7	0.017	7	0.040	6
C7	0.042	6	0.051	4-5-6	0.034	7

Like in the previous example, applied methods produced different weights of criteria. This is a consequence of prevailing preferences given to criteria *C*1, *C*2, and *C*3 by the decision-maker. In the case of criteria *C*1 and *C*3, AHP-L in artificially achieving full consistency of the pairwise comparison matrix (by following transition rule) gives two times value 21 which significantly surpasses the highest value from the ration scale, that is 9 – defined for absolute preference.

Remark #2.1: This example indicates possible insensitivity of ranking produced by fairly different methods. However, this is not the case if cardinal information is analyzed. The weights differ which is a clear consequence of the fact that in the case of AHP-S (real-life decision-making) the inconsistency index is CR=0.06 which is, although lower than tolerant value 0.10, still much higher than CR=0 in the case of AHP-L which generates fully consistent (but partly artificial) pairwise comparison matrix; the expression ‘partly’ relates to the fact that 6 comparisons above the main diagonal are real. Changes that might be produced if adjustments are implemented by the algorithm presented in [24] could make the comparison matrix more realistic, e.g. with entries corresponding to exact values from the ratio scale – to have semantic meanings. Adjustments are inherently unrealistic because there are many controversies regarding the issue of consistency, the readiness of decision-maker to accept corrections in his/her original judgments (as made in AHP-S), etc.

Remark #2.2: At this point worth commenting is the minimum error ε = 0.0676502 generated as the optimal value of the goal function by the LP program in the BWM. The structure of the LP program is such that ε is a unique tolerant difference between linear relations of all weights and their corresponding judgments in the comparison matrix. The optimal value ε in this example is not that high regarding size 7 of the matrix, and the set of weights can be considered as a sufficiently consistent solution. A conclusion might be that a low value of error ε may justify the decision to use BWM instead of AHP-S, or at least to either alternatively use the results of BWM only (once both methods are applied by the same decision-maker) or to combine the results from both methods, for instance by their geometric aggregation. This approach might be interesting especially for matrices of higher order.

3.4. Example #3 - Matrix size 8

Source papers: (a) [48] and (b) [49].

The third example is taken from two most recently published papers related to group evaluation of the following set of eight criteria applicable in ranking by importance protected natural areas in Serbia, recognized as Ramsar areas [50]: C1 – protection of habitats, C2 – biodiversity, C3 – extreme water regime, C4 – purposes, C5 – location, C6 – tourism and education; C7 – water quality, and C8 – socio-cultural heritage.

The results obtained by the three methods are presented in Tables 12-15.

Standard AHP

Table 12. AHP-S comparison matrix and derived criteria weights (Example #3)

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	Weight	Rank
C1	**1**	1/3	1/4	5	8	7	1/4	8	0.135	4
C2		**1**	1/2	5	8	7	1/2	8	0.200	3
C3			**1**	5	9	7	1	9	0.267	1-2
C4				**1**	5	3	1/5	5	0.060	5
C5					**1**	1/5	1/9	1	0.018	7-8
C6						**1**	1/7	5	0.041	6
C7							**1**	9	0.267	1-2
C8								**1**	0.018	7-8

Consistency measure: CR = 0.088 (<0.100; satisfactory)

Limited AHP

Table 13. AHP-L comparison matrix and derived criteria weights (Example #3)

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	Weight	Rank
C1	**1**	1/3	1/6	5/6	25/6	25/30	25/210	225/210	0.046	6
C2		**1**	1/2	5/2	25/2	25/10	25/70	225/70	0.137	3
C3			**1**	5	25	5	5/7	45/7	0.273	2
C4				**1**	5	1	1/7	9/7	0.055	4-5
C5					**1**	1/5	1/35	9/35	0.011	8
C6						**1**	1/7	9/7	0.055	4-5
C7							**1**	9	0.382	1
C8								**1**	0.042	7

Row #1: $a_{13}=a_{12}\cdot a_{23}=1/3\cdot1/2=1/6$; $a_{14}=a_{13}\cdot a_{34}=1/6\cdot5=5/6$; $a_{15}=a_{14}\cdot a_{45}=5/6\cdot5=25/6$
$a_{16}=a_{15}\cdot a_{56}=25/6\cdot1/5=25/30$; $a_{17}=a_{16}\cdot a_{67}=25/30\cdot1/7=25/210$;
$a_{18}=a_{17}\cdot a_{78}=25/210\cdot9=225/210$

Row #2: $a_{24}=a_{23}\cdot a_{34}=1/2\cdot5=5/2$; $a_{25}=a_{24}\cdot a_{45}=5/2\cdot5=25/2$;
$a_{26}=a_{25}\cdot a_{56}=25/2\cdot1/5=25/10$; $a_{27}=a_{26}\cdot a_{67}=25/10\cdot1/7=25/70$;
$a_{28}=a_{27}\cdot a_{78}=25/70\cdot9=225/70$

Row #3: $a_{35}=a_{34}\cdot a_{45}=5\cdot5=25$; $a_{36}=a_{35}\cdot a_{56}=25\cdot1/5=5$; $a_{37}=a_{36}\cdot a_{67}=5\cdot1/7=5/7$
$a_{38}=a_{37}\cdot a_{78}=5/7\cdot9=45/7$;

Row #4: $a_{46}=a_{45}\cdot a_{56}=5\cdot1/5=1$; $a_{47}=a_{46}\cdot a_{67}=1\cdot1/7=1/7$; $a_{48}=a_{47}\cdot a_{78}=1/7\cdot9=9/7$

Row #5: $a_{57}=a_{56}\cdot a_{67}=1/5\cdot1/7=1/35$; $a_{58}=a_{57}\cdot a_{78}=1/35\cdot9=9/35$

Row #6: $a_{68}=a_{67}\cdot a_{78}=1/7\cdot9=9/7$

BWM

Pairwise comparison vectors are defined as in Table 14.

Table 14. BWM vectors for best and worst criterion (Example #3)

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
Best criterion: **C7**	4	4	2	6	8	7	1	7
Worst criterion: **C5**	5	6	7	4	1	4	8	3

The LP problem for this setting is:

$$
\begin{gathered}
\min\ \varepsilon \\
\text{s.t.} \\
|w_7 - 4w_1| \le \varepsilon \\
|w_7 - 4w_2| \le \varepsilon \\
|w_7 - 2w_3| \le \varepsilon \\
|w_7 - 6w_4| \le \varepsilon \\
|w_7 - 8w_5| \le \varepsilon \\
|w_7 - 7w_6| \le \varepsilon \\
|w_7 - 7w_8| \le \varepsilon \\
|w_1 - 5w_5| \le \varepsilon \\
|w_2 - 6w_5| \le \varepsilon \\
|w_3 - 7w_5| \le \varepsilon \\
|w_4 - 4w_5| \le \varepsilon \\
|w_6 - 4w_5| \le \varepsilon \\
|w_8 - 3w_5| \le \varepsilon \\
w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 = 1 \\
w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8 \ge 0
\end{gathered} \tag{9}
$$

and the solution is: $w_1 = 0.1074169$; $w_2 = 0.1074169$; $w_3 = 0.2148338$; $w_4 = 0.07161125$; $w_5 = 0.03222506$; $w_6 = 0.06138107$; $w_7 = 0.3437340$; $w_8 = 0.06138107$; $\varepsilon = 0.0859335$.

In this example, value ε = 0.086 is sufficiently close to zero and the solution of model (9) can be considered as satisfactorily consistent.

Altogether (AHP-S, AHP-L, BWM)

Table 15 summarizes the result obtained by three methods.

Table 15. Summary of AHP-S, AHP-L, and BWM solutions (Example #3)

Criteria	Standard AHP (AHP-S)		Limited AHP (AHP-L)		Best-Worst Method (BWM)	
	Weight	Rank	Weight	Rank	Weight	Rank
C1	0.135	4	0.046	6	0.107	3-4
C2	0.188	3	0.137	3	0.107	3-4
C3	0.262	1-2	0.273	2	0.215	2
C4	0.066	5	0.055	4-5	0.072	5
C5	0.020	7-8	0.011	8	0.032	8
C6	0.048	6	0.055	4-5	0.061	6-7
C7	0.262	1-2	0.382	1	0.344	1
C8	0.020	7-8	0.042	7	0.061	6-7

The methods are in relatively good agreement regarding the final ranking of criteria, especially on the top three positions. The sum of the weights of criteria *C*7, *C*3, and *C*2 is 0.712 obtained by AHP-S. In the case of AHP-L, the sum is 0.792, and in BWM this sum is 0.666. The weights of the remaining five criteria derived by three methods are relatively small. As in the previous example, when the AHP-S synthesis of local weights of six Ramsar areas in the northern part of Serbia (known as the Vojvodina Province) across all criteria resulted in approximately the same order of Ramsar areas when the weights obtained by the AHP-L and the BW methods are used instead. In the decision processes of selecting the most important element of the hierarchy (alternative or criterion) these methodologies are suitable for use because the application of different methods shows the same best-ranked solution.

Applied methods expectedly produced different weights of criteria as a result of the difference in methodologies used while creating a comparison matrix. Note that AHP-L and BWM give a significantly higher value than AHP-S for the first ranked criterion *C*7. This difference is almost 50% higher in the case of AHP-L versus AHP-S, and 31% in the case of BWM versus AHP-S.

In this example, AHP-S required $n(n\text{-}1)/2=(8\cdot7)/2=28$ multiplicative preference relations, BWM required $2n\text{-}3=2\cdot8\text{-}3=13$ relations, and in the case of AHP-L, only 8 comparisons were required. Reduction in the number of comparisons used in AHP-L and BWM versus AHP-S by 54% and 71%, respectively, is furthermore significant while the result is not that much different regarding the ranking of top positioned criteria.

In presented examples for prioritizing 6, 7, and 8 criteria number of required comparisons was different regarding the prioritization method used. In all cases AHP-L required a minimum number of comparisons, more comparisons were needed if BWM is used, and finally, the number of comparisons was largest for AHP-S. Table 16 and Fig. 3 illustrate this effect of enlarging the information base with the size of the matrix. Note that if a matrix is of size 9, the number of comparisons drops by more than a half, actually near 60% if BWM is used instead of AHP-S. The number of comparisons does not significantly affect the final result because the best (e.g. top three) and the worst (e.g. last two) criteria will be obtained by using any method.

The number of comparisons could depend on the individual preferences of decision-makers. Depending on whether the decision-maker feels, he/she can express his/her opinion (1) comparing all the criteria with each other as is the case by using AHP-S, or (2) expressing right at the beginning which is the best and which is the worst criterion creating further opinions and comparison of other criteria according to the best/worst criterion (case of BWM).

Table 16. Relationships between methods AHP-S, AHP-L, and BWM regarding the number of comparisons required for different sizes of comparison matrices/compared elements

Size of matrix/ Number of compared elements *(n)*	Number of comparisons			R
	AHP-S $n(n-1)/2$	AHP-L $n-1$	BWM $2n-3$	BW/AHP-S = $(4n-6)/(n^2-n)$
2	1	1	1	1.00
3	3	2	3	1.00
4	6	3	5	0.83
5	10	4	7	0.70
6	15	5	9	0.60
7	21	6	11	0.52
8	28	7	13	0.46
9	36	8	15	0.42

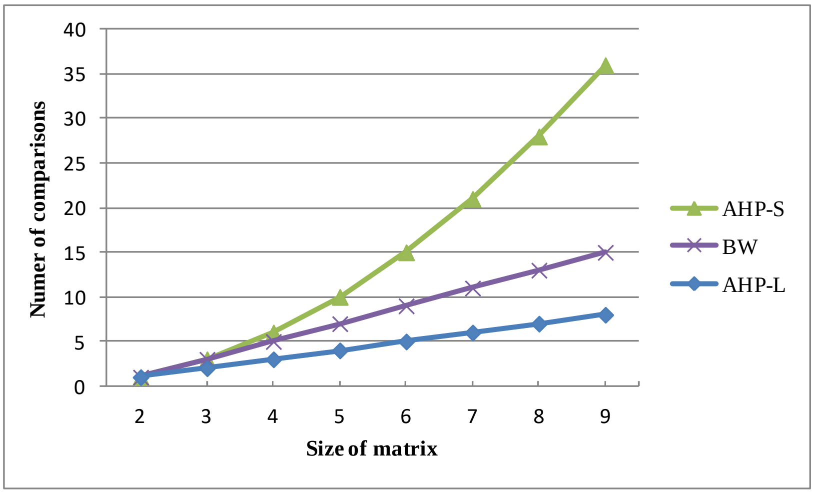

Fig. 3. The number of comparisons required by methods AHP-S. AHP-L and BWM for different sizes of comparison matrices

In the selection processes when the ranking of criteria or alternatives leads to the final choice(s) it would be desirable to choose a methodology for ranking according to the number of offered criteria and alternatives. For instance, if the size of the comparison matrix is up to 4 elements then the number of necessary comparisons is similar regardless of which method was used (Cf. Table 16). When it comes to larger comparison matrices (the number of elements is larger than 5), it is desirable to choose a methodology that will quickly and efficiently show reliable and accurate results. Decision matrices with 8 or more criteria should be avoided in the real application of multi-criteria methods due to the proven high inconsistency of the process of comparison and evaluation of criteria. In that case, one should try to reduce the number of criteria or simplify problem hierarchies by dividing the criteria into sub-criteria (dividing one level into two or three new levels). This procedure requires a larger number of comparison matrices and thus a larger number of evaluations. In that case, the method should be chosen which requires a smaller number of comparisons.

4. Conclusions

The analytic hierarchy process (AHP) is one of the most used methods for supporting decision-making processes. It is intuitively correct because it follows the usual human intent to decompose problems into smaller parts and determine the importance of its elements by putting them into a hierarchical structure. Local evaluations are performed by pairwise comparisons of elements at a given level versus adjacent elements in the upper level. Following the prioritization process and deriving the local weights of compared elements at all levels of the hierarchy, synthesis of local weights produces the final result: priorities of decision elements (usually alternatives) at the bottom level of hierarchy regarding stated goal at the top of the hierarchy.

The most common situation is that intermediate prioritization is performed for criteria set versus goal. The final result of AHP strongly depends on priorities obtained for criteria as decision elements of prime importance in making any decision. They are usually set at the first level of the hierarchy, positioned just below the goal as a global evaluation target to be met by the alternatives at the bottom level of the hierarchy. Priorities represented by weights of criteria play the most important role in achieving the goal because they moderate the remaining synthesis process, and therefore the prioritization method to be applied at this level is essential for the complete AHP application.

While the AHP-S and AHP-L are matrix methods, the third used method in this study was BWM, a strictly linear programming (optimization) method. The methods use a different number of judgments made by the decision-maker about the mutual importance of criteria as decision elements. Because of differences in methodology and information availability (judgments used), as a consequence, all three methods produced different weights of criteria in all three studied cases with matrices of sizes 6, 7, and 8. Consistency of the prioritization process was in all analyzed cases satisfactory, of course except AHP-L where consistency is absolute due to partly artificial judgments generated by transition rules. Furthermore, the AHP-L method is very sensitive to each of the elements considered where n-1 long path presents extremal among spanning trees.

Relatively large matrices are used to better contrast differences among the models' outcomes. In the majority of cases it was evidenced that although corresponding weights of each criterion are different, the ranking of criteria is generally the same. Adopting only top-ranked criteria and 'deleting' the others may lead to more focused manipulation of the decision-making process, especially in situations when group decisions should be made by reaching consensus or by applying some of the social choice (election) models.

In all analyzed cases applied methods used the same ratio scale, that is Saaty's scale with 17 discrete values from the set {1/9, 1/8, ... 1/2, 1, 2, ... 9}. Worth mentioning is that this scale is the only starting point in the AHP-L method because generated entries in the upper triangle of all analyzed matrices did not necessarily belong to this scale; in many cases generated entries are either within the scale range (1/9 - 9) but without clear semantic meaning, or outside the scale, again without meaning. Even in the case of this method, the results are similar to the results obtained by the other two methods, especially in ranking decision elements. Nevertheless, note that the mechanism used to create a matrix in AHP-L is dependent on the "seed a_{ij}" selected (the green highlighted cells), and it would be possible to use others but generate different results.

This study concludes that if compared with complete information used by AHP-S, BWM produces a sufficiently good vector of weights, despite the reduced information it relies on. Its result is on a 'safe side' within the decision-making framework in a sense that compensation for the slightly less trustful result than this produced by AHP-S can be found in less effort decision-maker or analyst must put while making judgments. For instance, in the case of the matrix of size 5, BWM requires 7, while AHP-S requires 10 judgments which is a reduction of 30%. In the case of the matrix of size 9, the corresponding number of judgments is 15 and 36, respectively, which is a reduction of 58%, more than a double. The AHP-L is inferior to the other two used methods because it is not semantically realistic, although it is 'heading' toward full consistency and relatively good results. Without fine-tuning to make generated entries in the comparison matrix to fit exact values from the scale (1/9 - 9), this method is not justified from a real-life usage standpoint.

Regarding recommendation which method to use, it is hard to draw a general conclusion. One of the decisive issues could be who is involved in the evaluation and calculation process. The AHP-S method, which has been widely recognized in the scientific community for years, is more flexible in a manner that today there is ready-made software that does not require background knowledge of mathematics and supplementary calculations, and it is suitable for wider use by expert groups. BWM is still a relatively new method for which there is no ready-made software and requires additional time and knowledge in the field of linear programming and the use of available mathematical software to calculate the mathematics of the method.

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