Probability-Model based
Network Traffic Matrix Estimation

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Abstract. Traffic matrix is of great help in many network applications. However, it is very difficult to estimate the traffic matrix for a large-scale network. This is because the estimation problem from limited link measurements is highly under-constrained. We propose a simple probability model for a large-scale practical network. The probability model is then generalized to a general model by including random traffic data. Traffic matrix estimation is then conducted under these two models by two minimization methods. It is shown that the Normalized Root Mean Square Errors of these estimates under our model assumption are very small. For a large-scale network, the traffic matrix estimation methods also perform well. The comparison of two minimization methods shown in the simulation results complies with the analysis.

Keywords: traffic matrix estimation, probability model, NRMSE.

1.   Introduction

In an IP network, the traffic matrix (TM) describes the traffic volumes traversing the network from the input nodes to the exit nodes over a measured period. Such a matrix is very helpful in many network applications such as capacity plan, anomaly detection, traffic engineering, and network reliability analysis [1]. In these application scenarios, TMs act as an important input. The outputs of various network engineering tasks are directly relevant to the input. So plenty of work aim to conduct measurements on TMs [5] or indirect inference from readily available link measurements [15, 16]. Whichever way is used, it is extremely hard to obtain the traffic matrix for a large network.

The link measurements which are related to traffic matrix is more realistic to be obtained in practice. Denote the link measurements by Y and the traffic matrix by X. Y is an n_l by 1 vector and X is n_p by 1 vector where $n_l << n_p$. Thus, the relationship between link measurements and the traffic matrix is represented by $Y = AX$ where A is

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the routing matrix. Given a set of link measurements, we aim to infer an overall view of the traffic matrix. This is a classic under-constrained, linear-inverse problem. we cannot obtain a solution if we do not introduce any side information such as the prior model of a traffic matrix. Therefore, paper [11, 14] assumed the origin-destination-demands follows a Poisson distribution. In [2], Origin-Destination (OD) pairs are modeled according to a Gaussian distribution. Medina et al. assumed a logit-choice model in [7] and M. Roughan et al. proposed a gravity model [9, 10].

There are plenty of methods proposed based on these prior traffic model assumption such as Bayesian inference, information theoretic approach, maximum likelihood estimation, Linear programming and multiresolution analysis method [12]. These methods perform very differently for different traffic data. There is no method which can work well on all traffic data. Compressed Sensing (CS) is recently greatly developed in many applications. It shows that any sufficiently compressible signal can be accurately recovered from a small number of non-adaptive, randomized linear projection samples [6]. It can also be applied in traffic matrix estimation due to the sparsity of the traffic matrix such as the work in [3, 4, 8, 17]. In this paper, we look for a sparsity model which can be used for efficient traffic matrix estimation, resulted in a probability model [13]. This model is simple but efficient. From the simulation results, we will see a generalized model based on our probability model can work well in Origin-Destination pair traffic estimation. This generalized probability-based model can not only help to generate traffic data for large-scale network simulations, but also help to study sparse traffic matrix.

2.   Model

A large-scale network usually involves multiple routers/switches in multiple levels. In each level, many routers/switches are located for redundancy and reliability. If we simplify the network topology without changing basic networking function, we will get a tree-like network. In the tree-like network, there is one parent node and N children. The parent node is considered as a node connecting outside Internet and the child nodes are servers which may be composed of a set of children.

Our model starts with a single star network. This star network is assumed to include one parent node and several child nodes which are connected through one router. This star may be the lowest level star and all of similar stars can be aggregated to form the original network. The traffic estimation on this simple star will be formed into the whole network traffic matrix. By doing so, estimating a $N * N$ traffic matrix by $2N$ link measurements which is highly under-constrained though, can be largely reduced in the dimensionality of the problem to a simple star network.

We assume a network model which is composed of n servers which are connected to the Internet node via 1 router. Given the link measurements Y observed on these edge links which are immediate link connecting the end nodes, the problem is to estimate the end-to-end traffic matrix X. $Y = AX$, A is an n_l by n_p routing matrix.

In the simple single star model we assume that all servers are the same. We will relax this assumption later on, but this allows us to build a very simple traffic model firstly. This model only needs three probability parameters which is thus named as the probability model. In our probability model, it is assumed any pairs of servers send traffic to each other with the same probability p_1, any server sends traffic to the Internet node with the

same probability p_2 and the Internet node sends the traffic to any server with the same probability p_3. Obviously,

$$(n-1)p_1 + p_2 = 1, np_3 = 1. \tag{1}$$

3. Traffic matrix estimation

Firstly, we consider a network composed of 2 servers, 1 node denoting Internet and 1 router as Figure 1 shows. Thus we have $p_1 + p_2 = 1$ and $p_3 = 1/2$. As we consider that a server will either send the packets to other servers or to the Internet node. There will not be any traffic to itself. Thus the routing matrix A is

$$A = \begin{pmatrix} 0\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0\,1\,0\,0 \\ 0\,0\,0\,1\,0\,1\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,0\,1\,1\,0 \\ 0\,0\,1\,0\,0\,1\,0\,0\,0 \end{pmatrix} \tag{2}$$
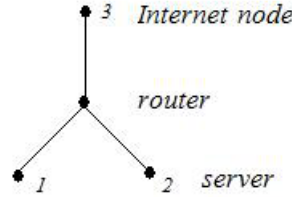

Fig. 1. A network with 2 servers and 1 Internet node

Assume the total traffic on each node are $X' = \{x'_1, x'_2, x'_3\}$, where $x_i, i \in \{1, 2, \ldots, n+1\}$ denotes the total traffic generated by node i. The elements in Y are the traffic on outgoing link from node 1, incoming link to node 1, and then node 2, and 3 with the same order.

Thus,

$$Y = BX' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & p_1 & p_3 \\ 0 & 1 & 0 \\ p_1 & 0 & p_3 \\ 0 & 0 & 1 \\ p_2 & p_2 & 0 \end{pmatrix} X' \tag{3}$$

Normally we don't have the full set of the measurements on Y. For example, we only have 3 links' measurements which is denoted by Y_s, $Y_s = B_s X'$. If we choose 3 links, for example, incoming link to node 1, incoming link to node 2, incoming link to Internet

node. We denote the selected link measurements as Y_s, and the selection matrix is B_s. Then we have

$$B_s = \begin{pmatrix} 0 & p_1 & p_3 \\ p_1 & 0 & p_2 \\ p_2 & p_2 & 0 \end{pmatrix} \tag{4}$$

Since $det(B_s) \neq 0$, we have $X' = B_s^{-1}Y_s$. Because the relationship between the OD traffic matrix X and traffic generating by end nodes X' can be described by the assumed probability matrix as follows.

$$X = PX' = \begin{pmatrix} 0 & 0 & 0 \\ p_1 & 0 & 0 \\ p_2 & 0 & 0 \\ 0 & p_1 & 0 \\ 0 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \\ 0 & 0 & p_3 \\ 0 & 0 & 0 \end{pmatrix} X' \tag{5}$$

Therefore we have $X = PB_s^{-1}Y_s$. P and B_s can both be calculated as above, we can get a solution to X in this probability model and let it be X_p.

$$X_p = \begin{pmatrix} 0 & 0 & 0 \\ \frac{-1}{2} & \frac{1}{2} & \frac{p_1}{2p_2} \\ \frac{-p_2}{2p_1} & \frac{p_2}{2p_1} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \frac{p_1}{2p_2} \\ 0 & 0 & 0 \\ \frac{p_2}{2p_1} & \frac{-p_2}{2p_1} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{-p_1}{2p_2} \\ \frac{1}{2} & \frac{1}{2} & \frac{-p_1}{2p_2} \\ 0 & 0 & 0 \end{pmatrix} Y_s \tag{6}$$

Therefore, we can obtain a solution related to one simple parameter under the assumption of the probability model. If p_1 can be set an appropriate value according to the realistic networks, Y_s is observed measurements on selected links, we can have a determined X_p.

Now our problem is to find a solution X which is the closest to X_p under the constraint $Y = AX$. That is,

$$Minimize \ \ f(X) = \parallel X - X_p \parallel, \ \ s.t. \ \ Y = AX,$$

where $\parallel \cdot \parallel$ is the L_2 norm. For simplicity in comparison, we call this to be Minimization 1 (Min 1) method.

This method is difficult to find an efficient solution if there are errors in measurements on Y. Therefore, we can employ Lagrangian function. The aim is thus to minimize $L(X, \lambda)$.

$$L(X, \lambda) = \parallel X - X_p \parallel + \lambda \parallel AX - Y \parallel \tag{7}$$

Again, for simplicity in comparison, we denote this by Min 2 method. In Min 2 method, L_2 norm is not easy to write in the form of quadratic X and using derivative, we solve the problem of

$$Minimize \ g(X) = \parallel X - X_p \parallel^2, \ \ s.t. \ Y = AX,$$

and construct the Lagrangian function as follows.

$$L(X, \lambda) = \parallel X - X_p \parallel^2 + \lambda \parallel AX - Y \parallel^2 \tag{8}$$

Thus,

$$\begin{aligned} L(X, \lambda) = \ & \lambda X^T A^T AX + \lambda Y^T Y - 2\lambda Y^T AX \\ & + X^T X - 2X^T X_p + X_p^T X_p \end{aligned} \tag{9}$$

The Karush-Kuhn-Tucker conditions for this local minimum are given as follows.

$$\frac{\partial L(X, \lambda)}{\partial X} \geq 0, \tag{10}$$

$$\frac{\partial L(X, \lambda)}{\partial \lambda} = 0, \tag{11}$$

$$X\frac{\partial L(X, \lambda)}{\partial \lambda} = 0, \tag{12}$$

$$\lambda\frac{\partial L(X, \lambda)}{\partial \lambda} = 0, \tag{13}$$

and

$$X \geq 0, \lambda \geq 0.$$

Deriving from above, the KKT conditions are simplified into:

$$(\lambda A^T AX + I)X = \lambda A^T Y + X_p, \tag{14}$$

$$X^T A^T AX - 2Y^T AX + Y^T Y = 0, \tag{15}$$

and

$$X \geq 0, \lambda \geq 0.$$

Hereby we can solve this Minimization problem by quadratic programming. Intuitively, in this quadratic programming, the more links' measurements are considered, the better the result is obtained for $\widehat{X}$.

4. Simulations

4.1. Metrics

As we discussed in the above section, there are two regularization methods which obtain the traffic matrix closest to the matrix X_p in the prior assumed model (also called the probability model). To compare these two methods, we will use Normalized Root Mean Square Error (NRMSE) to show the performance of estimated traffic matrices. Here the NRMSE is defined as e.

$$e = \frac{\sqrt{\sum_i (\hat{x}_i - x_i)^2 / (N \cdot (N+1))}}{Max(x_i) - Min(x_i)}. \tag{16}$$

Since quadratic distances are acceptable distance metric and lead to simple quadratic optimizing problem, it is easy to obtain solutions to both minimization problems. As we have denoted for simplicity, the Minimization problem $Min\{d(\hat{X}, X_p)\}$ subject to $Y = AX$ and $X \geq 0$ is denoted by Min 1 method and the other Minimization problem by Min 2 method.

Firstly, we simulate both methods on a network with $N = 8$ servers and one Internet node. Thus in total $2(N+1) = 18$ links measurements are available. The traffic matrix to estimate is a 9 by 9 matrix, i.e. an 81 by 1 vector. We obtain a cumulative distribution function (CDF) of NRMSE to observe the performance of these estimates. We then have a look at the sensitivity of NRMSEs to λ. As the number of links used in measurements increases, we will check the trend of NRMSEs.

4.2. A General Model

The simple prior model leads to all servers having identical traffic. This may not be accordance with the practical cases where there are differences on measurements to different servers. We thus generalize the above simple model to a more practical model which combines this simple model with a random traffic model. This general model is described to be

$$X_{gen} = \alpha X_p + (1 - \alpha) X_{random}.$$

In this general model, it is seen that the parameter α defines a random traffic matrix to be estimated which is still related to the prior assumed probability model. The changes of α give different weights of the prior model in this general model. Intuitively, we can conclude that the performance of estimates would be affected by this parameter α. Our simulation will check how NRMSE differs as α increases. We also introduce a bias factor β which counts the ratio of wrong measurements to correct measurements. This factor is helpful in testing the performance of estimation in both the probability model and the general model.

4.3. Experiments

The Figure 2(a) is obtained by monitoring 10 links' measurements. The other two important parameters are set as $a = 0.5$ and $\lambda = 10$. We checked the CDF of NRMSE of the

estimates by two minimization methods in two models separately. As shown in Figure 2(a), the CDF of NRMSE of estimated results by Min 2 method (represented by two red curves) are worse than that by Min 1 method. However Min 2 method gives very small NRMSE whose CDF approaches to 1 with an NRMSE of 0.045 in the general model. In the probability model, both methods work very competitively and Min 1 is only a bit better than Min 2 method. We also note that by using Min 2 method, the estimation process gives a much better performance in the probability model than in the general model. Min 1 method does not show this superiority in probability model to general model which shall be shown in a more large-scale network.

Figure 2(b) shows the results when using the full set of 18 links to estimate the original traffic matrix. It is obvious both methods work out CDFs approaching to 1 with smaller NRMSEs, especially when we look at the performance of Min 2 method in the general model. In the probability model, Min 2 method works slightly better than Min 1 method. This is because more links measurements give more constraints in $Y = AX$ which limits the searching space of finding a minimal distance from $\hat{X}$ to X_p. This outperformance will be more obvious when there are errors in measurements. This also gives the reason that we prefer Min 2 method to Min 1 method in a large-scale network with more possible errors in measurements.

Now we have a look at how NRMSE varies as λ increases when using Min 2 method. Again, we use measurements on 10 links in this 9-node network simulation and let $\alpha = 0.5$. Figure 3 shows that the NRMSE does not change when λ takes a value greater than a threshold. It is found λ does not affect the estimation performance if it is set a value greater than 5. Therefore, the value of λ being 10 in above simulations provides a good initialization for the traffic estimation process to run.

In the general model, α is a parameter which adjusts the weights of the traffic data under the probability model assumption and the random traffic data. Figure 4(a) shows how this parameter affects the performance of estimation. It is obvious that as α increases which means the random data weights less, both methods work better. In Figure 4 (a), since all measurements are correct, both methods can work out the same TM which is closest to the practical TM. The curves of both methods are overlapped. When there are errors in measurements, two methods have different performances. We set the bias factor to be 0.1 in Figure 4(b), that is, there are 10% links which have errors in measurements. Min 2 method shows a better performance than Min 1 method. When the bias factor increases which means there are more errors in measurements, the outperformance of Min 2 method to Min 1 method shall be greater.

In Figure 5, the read curve denoted by data 1 gives the result of Min 2 method in the general model. As the number of links used in estimation increases, data 1 shows an improved estimation performance of Min 2 method in the general model. The other three cases do not show obvious change as the number of links used in estimation varies. It is predicted in a large-scale network, using more link measurements would be more beneficial except the case where Min 1 method is used and errors in measurements exist.

In practice, the network size is much larger than the above 9-node network simulated above. How will these two estimation methods perform as the network size N is increasing? Figure 6 gives the normalized RMSE by two methods in two models as N changes. We let $\alpha = 0.5$, $\lambda = 10$ and the number of links used to estimate be three. Network size is changing from 4 nodes to a reasonably large size 53 nodes. It is seen that the NRMSE
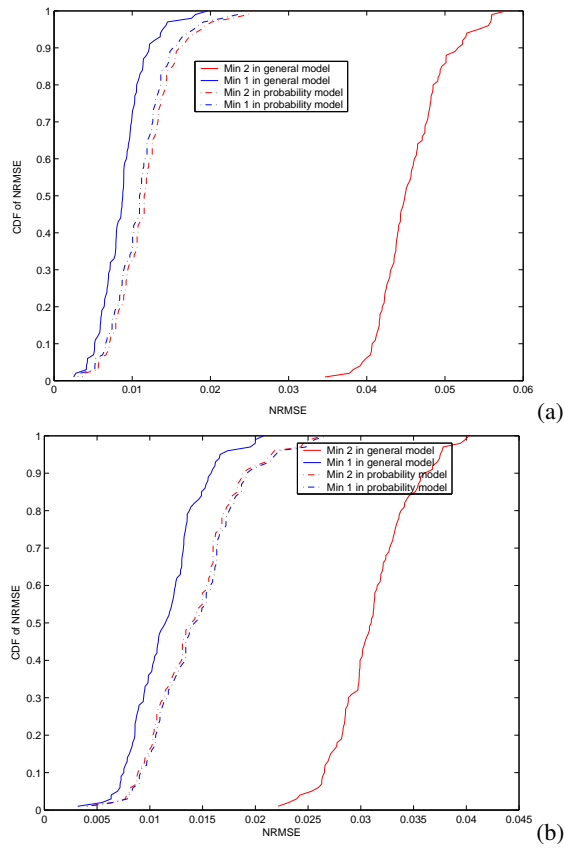
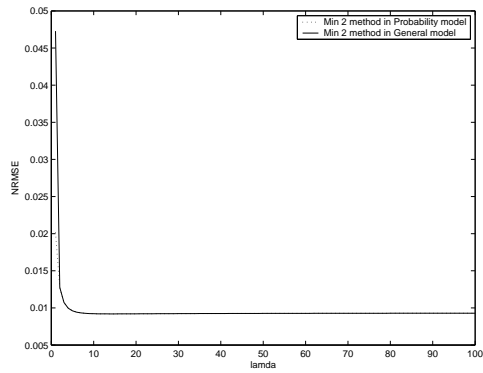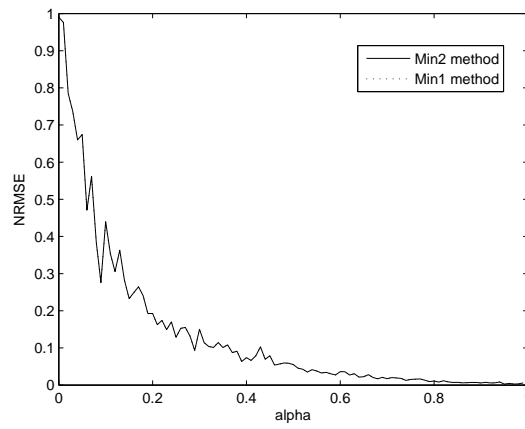**Fig. 2.** CDF of Normalized Root Mean Square Error by using both Minimization methods for two models in a 9-node network
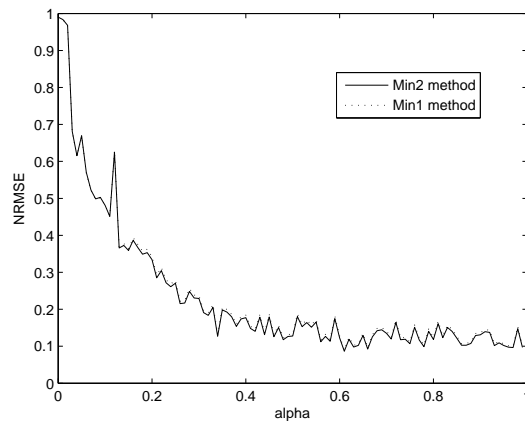

Fig. 3. Normalized Root Mean Square Error as λ changes by Min 2 method in two different models

(a)

(b)

Fig. 4. CDF of Normalized Root Mean Square Error by using both Minimization methods for two models in a 9-node network
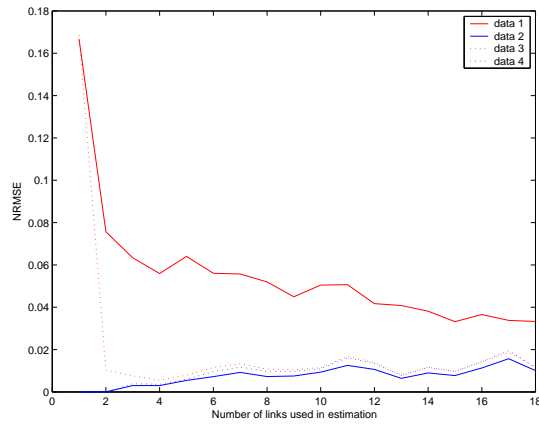
Fig. 5. Normalized Root Mean Square Error as the number of links used in estimation varies by two methods in two different models

trends to be smaller as the network size increases. Therefore, we can predict when the network size is greater, say hundreds or thousands, the NRMSE of these two estimation methods would work fairly well. Figure 6 also shows two methods work better in probability model than in general model. As we only use 3 links to estimate the total traffic matrix, Min 1 method performs a bit better than Min 2 method. When we have more links' measurements, Min 2 will show its superiority to Min 1 method as shown in above simulation. Note also both methods work well and are convergent when only 3 links are used in estimation. When there are more links' measurements available, both methods will perform better.
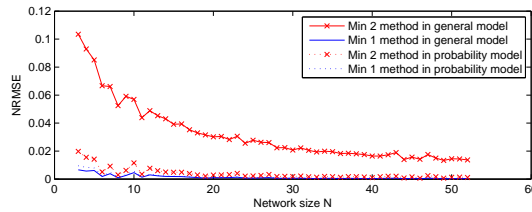

Fig. 6. Normalized Root Mean Square Error as N changes by two methods in two different models

5.   Conclusion

Traffic matrices in a large-scale network are difficult to be estimated because the estimation problem from limited link measurements is highly under-constrained. We propose a simple probability model for a star-like network which is easy to be aggregated to more

star-like networks together to form a large-scale practical network. In order to enable the probability model to adapt to more realistic data in the network, we generalize the probability model to include the random traffic data. Traffic matrix estimation is then conducted under these two models by two minimization methods. It is shown that the Normalized Root Mean Square Errors of these estimates under our model assumption are very small. For a large-scale network, the traffic matrix estimation methods also perform well. The comparisons between two minimization methods showed that Min 2 method is more robust to network measurements errors and thus preferred in practice.

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