

# Background Modeling from Video Sequences via Online Motion-Aware RPCA

Xu Weiyao<sup>1</sup>, Xia Ting<sup>1</sup>, and Jing Changqiang<sup>2\*</sup>

<sup>1</sup> Zaozhuang University

Zaozhuang 277160, Shandong Province, China

{xuweiyao\_2008,xiaiyuxue121}@126.com

<sup>2</sup> Linyi University

Linyi 276000, Shandong Province, China

jingchangqiang@lyu.edu.cn

**Abstract.** Background modeling of video frame sequences is a prerequisite for computer vision applications. Robust principal component analysis(RPCA), which aims to recover low rank matrix in applications of data mining and machine learning, has shown improved background modeling performance. Unfortunately, The traditional RPCA method considers the batch recovery of low rank matrix of all samples, which leads to higher storage cost. This paper proposes a novel online motion-aware RPCA algorithm, named OM-RPCAT, which adopt truncated nuclear norm regularization as an approximation method for of low rank constraint. And then, Two methods are employed to obtain the motion estimation matrix, the optical flow and the frame selection, which are merged into the data items to separate the foreground and background. Finally, an efficient alternating optimization algorithm is designed in an online manner. Experimental evaluations of challenging sequences demonstrate promising results over state-of-the-art methods in online application.

**Keywords:** Computer vision, Background modeling, Online RPCA, Truncated nuclear norm.

## 1. Introduction

Background modeling aims to extract foreground objects from videos, which has been widely used in many fields, such as object detection [46][29][18], object localization [33], and image alignment [28]. It aims to initialize efficient and accurate background modeling from a series of video frames.

Many background modeling methods have been proposed in the past few years. Recently, RPCA [2][17][4]has attracted wide attention in the fields of video surveillance and computer vision, which are based on decomposition of the matrix into sparse and low rank components, and has shown improved performance in background modeling[43]. RPCA decomposes the observed video frame matrix into background and foreground[7][16].

Let  $Z \in R^{m \times n}$  be the observational data, which can be represented in matrix form. RPCA attempts to decompose  $Z$  as the sum of a sparse matrix  $F$  and a low rank matrix  $B$ .

$$\min \text{rank}(B) + \lambda \|F\|_0 \quad \text{s.t.} \quad Z = B + F \quad (1)$$

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\* Corresponding author

where  $\|*\|_0$  denotes  $l_0$  norm, i.e., the number of nonzero elements, which is the regularization term for promoting sparsity.  $\lambda$  is a regularization parameter. Unfortunately, due to the discontinuity of the rank function and the nonconvexity of  $l_0$  norm, the optimization problem (1) is ordinarily NP-hard. Many researchers are seeking suitable alternatives to rank functions[8][6]. In addition,  $l_0$  norm can convert problem (1) into a convex optimization problem. Wright et al. [2] proved that while the sparse matrix  $F$  is sufficiently sparse, the low-rank matrix  $Z$  can be recovered by solving the following problem:

$$\min \|B\|_* + \lambda\|F\|_1 \quad \text{s.t.} \quad Z = B + F \quad (2)$$

where  $\|B\|_*$  is the nuclear norm of  $B$ .

The problem (2) is a convex optimization problem, and many algorithms have been proposed for this problem. Unfortunately, most of the algorithms solve this problem in a batch manner. Because all samples loaded in memory during the optimization procedure may have high storage costs, it is especially unacceptable for large-scale sample sets. In addition, while all the samples are collected by the streamlined way, these algorithms cannot update the low dimensional subspace efficiently while a new sample is adding. Each iteration needs to optimize every frame, which may seriously limit the scalability of streaming video.

To solve these problems, the online method of RPCA has recently been proposed. Many online mode algorithms for background modeling have been pursued. Shen et al. [30] adopted max norm as a substitute of the rank function in problem (2) to solve the RPCA problem in an online method.

However, the background in dynamic sequences may include multiple motions, which makes the accurate modeling more challenging. In most previous methods, objects are not moving. Smearing artifacts would be introduced while dealing with slow motion and stationary foregrounds. To be aware of motion, when the background scenes change gradually, many RPCA methods exhibit degraded performance, such as under changing lighting conditions. Moreover, existing methods such as Zhou et al. [46] produced an overwhelming outlier in the low-rank component, when the background was heavily occluded by foreground objects. Javed et al. [15] created a background model by using a modified version of RPCA to generate a low-rank matrix from a set of matrices.

Recently, X. Ye [42] proposed a motion-assisted matrix restoration (RMAMR) model for the separation of background objects from the foreground objects, in which the dense motion fields were incorporated into the framework of the RPCA. J. Yang [40] proposed an online motion-assisted RPCA model for back ground recovery from video sequences. This method is more efficient for memory and is scalable for the long video sequences, which are weighted by motion information.

In Hu [11], a novel norm called the truncated nuclear norm was proposed. The new nuclear norm is subtracted from the sum of several maximal singular values, achieving better rank approximation. Based on this method, F. Cao [3] proposed a new algorithm which was called low-rank and sparse decomposition based on the truncated nuclear norm(LRSD-TNN). B. Hong [10] proposed a novel and online robust principal analysis algorithm via truncated nuclear norm regularization and designed an online optimization scheme in which the matrices were updated alternately.

W. Hu [12] analyzed the problem of mocap data completion based on the truncated nuclear norm. In order to reduce the redundant frames, a simple joint motion detection

and frame selection operation was adopted[31]. Unfortunately, these methods deal with batch data only. In order to meet the needs of dynamic subspace, Hu et al.[5] Proposed an online optimization method to deal with the static camera background scene in the video sequence of each sampling point.

Pan et al.[27] proposed a motion-assisted RPCA model based on matrix factorization, and designed an effective linear alternating direction multiplier method and matrix factorization algorithm to solve the proposed FM-RPCA model. Hu et al.[13] Proposed a non-convex rank approximation RPCA model based on segmentation constraints. Firstly, the original video sequence is divided into three parts by low rank matrix decomposition. Then, a new non-convex function is proposed to constrain the low rank feature[9].

In this paper, we propose an online motion-aware RPCA with a truncated nuclear norm regularization (OM-RPCAT) framework for background modeling. The key idea is to extend the motion-aware low rank matrix approximation methods into an online model. In addition, to get better effect, we added a joint method of frame selection. Motion information from the video sequence is estimated by two methods in this paper. This method replaces the objective of rank minimization by minimizing the truncated kernel norm, which can be represented in a matrix factorization form. Then, we designed an efficient iterative optimization method for implementation [39][14].

The rest of this paper is organized as follows: In Section II, we present the OM-RPCAT scheme and two methods for obtaining the motion estimation matrix. In Section III, we design an efficient optimization algorithm to solve the optimization function. In Section IV, our algorithm is evaluated by experiments. Conclusions are made in Section V.

## 2. Background Modeling via OM-RPCAT

In this section, the OM-RPCAT model is proposed for background recovery. The main idea of our method is to make a more rigorous approximation to the rank operator and to exploit an online method for solving the optimization problem. Moreover, the background of recovery always suffers from smearing artifacts in areas covered by slow-moving objects. In order to overcome this defect, we combine the motion information and frame selection into the framework to separate the background from the moving objects[19][22][21].

### 2.1. The Proposed OM-RPCAT Model

In particular, let  $Z \in R^{m \times n}$  be the observational data, with  $Z = (z_1, \dots, z_n)$ , and each  $z_i$  expresses a sample. Our goal is to decompose the matrix  $Z$  into the low-rank matrix and the sparse matrix. The traditional methods of recovering the two components  $B$  and  $F$  is solved by solving the following equation:

$$\min \frac{1}{2} \|Z - B - F\|_F^2 + \lambda_1 \|B\|_* + \lambda_2 \|F\|_1 \quad (3)$$

The truncated nuclear norm minimization is adopted as a more rigorous low-rank constraint on  $B$ . Therefore, the objective function for this method becomes:

$$\min \frac{1}{2} \|Z - B - F\|_F^2 + \lambda_1 \|B\|_T + \lambda_2 \|F\|_1 \quad (4)$$

where  $\|\cdot\|_F$  is the Frobenius norm,  $\|\cdot\|_T$  is the truncated nuclear norm,  $\|\cdot\|_1$  is the  $l_1$  norm,  $\lambda_1$  and  $\lambda_2$  are the regularization parameters.

To solve the problem in (4), we usually use iterative optimization methods, such as the augmented Lagrangian multiplier [20] or the accelerated proximal gradient. Unfortunately, these methods are implemented in batch processing. Hence, huge data storage costs are incurred when large data are solved. To overcome this problem, we factorize  $B$  as  $B=LR^T$ .

Given a matrix  $B$ , the relationship [11] between  $\|B\|_T$  and  $\|B\|_*$  is:

$$\begin{aligned} \|B\|_T &= \|B\|_* - \max Tr(UBV^T) \\ &, UU^T = I, VV^T = I \end{aligned} \tag{5}$$

where  $Tr(\cdot)$  denotes the trace of the matrix and  $I$  stands for the identical matrix.

Then, the nuclear norm can be factorized as follows [30]:

$$\|B\|_* = \min_{B=LR^T} \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) \tag{6}$$

where  $L \in R^{m \times d}$ ,  $R \in R^{n \times d}$ .

From this paper B. Hong [10], we can obtain the following relationship:

$$\begin{aligned} \|B\|_T &= \|B\|_* - \sum_{i=1}^T \sigma_i(B) \\ &= \frac{1}{2}\|L\|_F^2 + \frac{1}{2}\|R\|_F^2 - Tr(UBV^T) \end{aligned} \tag{7}$$

Thus, the problem(4) can be transformed into the following constrained problem:

$$\begin{aligned} \min & \frac{1}{2}\|Z - LR^T - F\|_F^2 + \\ & \lambda_1(\frac{1}{2}\|L\|_F^2 + \frac{1}{2}\|R\|_F^2 - Tr(ULR^TV^T)) + \lambda_2\|F\|_1 \\ \text{s.t.} & UU^T = I, VV^T = I \end{aligned} \tag{8}$$

Our OM-RPCAT model is proposed to separate the foreground and background by joining motion information in the video sequences. We introduce the matrix  $W$  to represent motion information,  $W \in [0, 1]$ . The elements in the matrix  $W$  indicate whether the pixels in  $Z$  belong to the background. Therefore, the final form of the problem is

$$\begin{aligned} \min & \frac{1}{2}\|W \circ (Z - LR^T - F)\|_F^2 + \\ & \lambda_1(\frac{1}{2}\|L\|_F^2 + \frac{1}{2}\|R\|_F^2 - Tr(MR^T)) + \lambda_2\|F\|_1 \\ \text{s.t.} & UU^T = I, VV^T = I \end{aligned} \tag{9}$$

where  $\circ$  denotes the Hadamard product, which is known as the element-wise product. Let  $M=V^TUL$ , and we use the fact that  $Tr(XYZ)=Tr(ZXY)$ :

$$Tr(ULR^TV^T) = Tr(V^TULR^T) \tag{10}$$

For each sample  $z_i$ , we can get the approximate value of each sample under dictionary  $L$  through  $Lr_i + f_i$ . The problem (9) can be decomposed into the sample form:

$$\begin{aligned} & \min \frac{1}{2} \|w_i \circ (z_i - Lr_i - f_i)\|_2^2 + \\ & \lambda_1 \left( \frac{1}{2} \|L\|_F^2 + \frac{1}{2} \sum_{i=1}^n \|r_i\|_2^2 - \sum_{i=1}^n m_i^T r_i \right) + \lambda_2 \sum_{i=1}^n \|f_i\|_1 \quad (11) \\ & \text{s.t. } UU^T = I, VV^T = I \end{aligned}$$

To simplify this problem, we introduce the  $l(z_i, L)$  function:

$$\begin{aligned} l(z_i, L) = & \min \frac{1}{2} \|w_i \circ (z_i - Lr_i - f_i)\|_2^2 + \\ & \frac{\lambda_1}{2} \|r_i\|_2^2 - \lambda_1 m_i^T r_i + \lambda_2 \|f_i\|_1 \quad (12) \end{aligned}$$

The problem (11) can be solved by minimizing the following loss function:

$$f_n(L) = \frac{1}{n} \sum_{k=1}^n l(z_k, L) + \frac{\lambda_1}{2n} \|L\|_F^2 \quad (13)$$

where  $z_i$  is the current frame,  $r_i$  represents the coefficient under the dictionary  $L$ , and  $w_i$  is the  $i$ th row of  $W$ .  $l(z_i, L)$  is the loss function for each sample.

## 2.2. Motion Estimation

In this section, the motion matrix  $W$  is generated by computing the video sequence  $Z$ . We use two methods to construct the weighting matrix  $W$ . The first method is to adopt the optical flow [1], and the second method involves joining the motion detection and frame selection [31][34][35][36].

**A.** The first method involves using optical flow to obtain the motion estimation matrix.

Assume that  $z_i$  and  $z_{i+1}$  are the two consecutive frames of a sequence  $Z$ . Then, the horizontal  $V^x$  and vertical  $V^y$  motion vector components can be obtained by estimating the optical flow field. The motion map  $W$  is constructed as follows[44][45]:

$$w_{i,j} = \begin{cases} 0, & \text{if } \sqrt{(v_{i,j}^x)^2 + (v_{i,j}^y)^2} \geq \tau \\ 1, & \text{otherwise} \end{cases} \quad (14)$$

where  $\tau$  is the threshold of motion magnitude according to the average intensity of the motion field determined experimentally.  $v_{i,j}^y$  and  $v_{i,j}^x$  are entries of  $V^y$  and  $V^x$  in the vertical motion fields and horizontal motion fields respectively.

**B.** The second method is to add the frame selection.

This scheme aims to reduce the number of redundant frames [31]. The index of the relevant frames is given as follows:

$$y_i = \begin{cases} 1, & \text{if } |\hat{z} - \hat{\mu}| \geq \tau \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

where  $\hat{\mu}$  denotes the mean value of the vector  $\hat{z}$  which is gradiented and regularized for each frame [31].  $\tau$  controls the threshold. When the label  $y_i$  is 1, the corresponding frame

will be selected. Due to the removal of invalid data frames, this method can restore the background of the video more accurately[23][24][26].

While the frame selection process is complete, the motion estimation of the selected frames is determined by:

$$w_k(i, j) = \begin{cases} 0, & \text{if } \frac{1}{2}(D_k(i, j))^2 \geq \beta \\ 1, & \text{otherwise} \end{cases} \quad (16)$$

Differing from this method [31],  $\beta$  is the thresholding parameter,  $D_k$  is the difference between the two consecutive frames, and  $z_t$  denotes the t-th frame:

$$D_t = \sqrt{(Z_t - Z_{t-1})^2} \quad (17)$$

### 3. Optimization Method.

We propose an algorithm to minimize the average cost, which could solve our OM-RPCAT model in detail. The coefficient  $r_t$ , sparse error  $f_t$ , motion estimation matrix  $w_t$ , and basis L are optimized using alternative methods. The optimization procedure is divided into two steps[37][38][41]:

In the first step, we optimize coefficient  $r_t$ , sparse error  $f_t$  and motion estimation matrix  $w_t$ . The  $w_t$  has been illustrated in Section 2.2.

$$\{r_t, f_t\} = \arg \min \frac{1}{2} \|w_i \circ (z_t - L_{t-1}r - f)\|_2^2 + \lambda_1 \left( \frac{1}{2} \|r\|_2^2 - m_t^T r \right) + \lambda_2 \|f\|_1 \quad (18)$$

where  $w_i$  denotes the motion map for the current frame  $z_t$ .

Update  $r_t$ : Given the current basis L, the coefficient  $r_t$  can be obtained by the following formula:

$$f(r) = \frac{1}{2} \|w_i \circ (z_t - L_{t-1}r - f_t^k)\|_2^2 + \lambda_1 \left( \frac{1}{2} \|r\|_2^2 - m_t^T r \right) \quad (19)$$

Let  $\partial f / \partial r = 0$ , and we can obtain the following solution:

$$r_t^{k+1} = (L_{t-1}^T \hat{W}_t^T \hat{W}_t L_{t-1} + \lambda_1 I)^{-1} (L_{t-1}^T \hat{W}_t^T \hat{W}_t (z_t - f_t^k) + \lambda_1 m_t) \quad (20)$$

where  $\hat{W}$  is a diagonal matrix formed by placing the elements on the diagonal.

Update  $f_t$ : The objective formula to optimize  $f_t$  induced from problem(18) is:

$$g(f) = \frac{1}{2} \|w_i \circ (z_t - L_{t-1}r_t^{k+1} - f)\|_2^2 + \lambda_2 \|f\|_1 \quad (21)$$

We used the common approach presented in E. J. Candes [28] to solve this problem and obtained the following closed formula:

$$f_t^{k+1} = S_{\lambda_2} [w_i \circ (z_t - L_{t-1}r_t^{k+1})] \quad (22)$$

where  $S_\tau[*]$  is a shrinkage operator, which is defined as

$$S_\tau[x] = \text{sign}(x) \max(|x| - \tau, 0) \quad (23)$$

In the second step, we optimize the basis matrices  $L_t$ ,  $V_t$ , and  $U_t$  under the previously obtained  $r_k$ ,  $f_k$ , and  $w_k$ . Based on problem(9), the objective function is derived as follows:

$$\begin{aligned} \{L_t, U_t, V_t\} = \min \frac{1}{2} \|W \circ (Z - LR^T - F)\|_F^2 + \\ \lambda_1 \left( \frac{1}{2} \|L\|_F^2 - \text{Tr}(MR^T) \right) \end{aligned} \quad (24)$$

Because of the Hadamard product, it is difficult to calculate the matrix multiplication for this formula. We introduce an additional variable Y:

$$Y_k = Z_k - L_k R_k^T - F_k \quad (25)$$

The above equation is converted to the following equation:

$$\begin{cases} Y_k^{l+1} = \arg \min_Y \frac{1}{2} \|W_k \circ Y_k^l\|_F^2 + \lambda_1 \left( \frac{1}{2} \|L_k^l\|_F^2 - \right. \\ \left. \text{Tr}(M_k R_k^T) \right) + \frac{\lambda_3}{2} \|Y_k^l - Z_k + L_k^l R_k^T + F_k\|_F^2 \\ L_k^{l+1} = \arg \min_L \frac{1}{2} \|W_k \circ Y_k^{l+1}\|_F^2 + \lambda_1 \left( \frac{1}{2} \|L_k^l\|_F^2 - \right. \\ \left. \text{Tr}(M_k R_k^T) \right) + \frac{\lambda_3}{2} \|Y_k^{l+1} - Z_k + L_k^l R_k^T + F_k\|_F^2 \end{cases} \quad (26)$$

where  $\lambda_3$  is a constrained parameter. After removing the irrelevant items, we can get the following:

Update Y:

$$\begin{aligned} Y_k^{l+1} = \arg \min_Y \frac{1}{2} \|W_k \circ Y_k^l\|_F^2 + \\ \frac{\lambda_3}{2} \|Y_k^l - Z_k + L_k^l R_k^T + F_k\|_F^2 \end{aligned} \quad (27)$$

We can compute Y in a pixel-wise manner:

$$y_{ik} = \frac{\lambda_3}{\lambda_3 + w_{ik}^2} (z_{ik} - f_{ik} - l_i r_k) \quad (28)$$

where  $l_i$  denotes the  $i$ th row of matrix L.

Update L:

$$\begin{aligned} L_k^{l+1} = \arg \min_L \lambda_1 \left( \frac{1}{2} \|L_k^l\|_F^2 - \text{Tr}(M_k R_k^T) \right) + \\ \frac{\lambda_3}{2} \|Y_k^{l+1} - Z_k + L_k^l R_k^T + F_k\|_F^2 \end{aligned} \quad (29)$$

where  $M = V^T U L \in R^{n \times d}$ .

The matrix L can be updated via the closed-form solution of the least square problem in Eq.(29):

$$L_k = (\lambda_3 (Z - F - Y) R + U^T V R) (\lambda_1 I + \lambda_3 R^T R) \quad (30)$$

$U_t$  is optimized by the following formula [10]:

$$U_t = \arg \max \text{Tr}(U L_t R_t^T V_{t-1}^T) \text{ s.t. } U U^T = I \quad (31)$$

Similarly,  $V_t$  is optimized by the following formula [10]:

$$V_t = \arg \max \text{Tr}(VR_t L_t^T U_t^T) \text{ s.t. } VV^T = I \quad (32)$$

The algorithm is shown as Algorithm 1.

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**Algorithm 1: OM-RPCAT Algorithm**

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**Input:**  
The observed data:  $Z = [z_1, z_2, \dots, z_n] \in R^{m \times n}$   
matrix:  $L_0 \in R^{m \times d}, U_0 \in R^{s \times m}, V_0 \in R^{s \times n}$   
regularization parameters:  $\lambda_1, \lambda_2, \lambda_3 \in R$   
motion matrix:  $W \in R^{m \times n}$   
number of frames:  $t$   
**Output:**  $L_n, R_n$

- 1 **for**  $t = 1$  to  $n$  **do**
- 2     coefficient  $r_t = 0$
- 3     sparse error  $f_t = 0$
- 4      $m_t$  is the  $t$ -th row of  $V_{t-1}^T U_{t-1} L_{t-1}$
- 5     compute motion matrix  $w_t$  by Eq.(14) or Eq.(16)
- 6     Stage 1: compute  $r_t$  and  $f_t$
- 7     **while** *not converged* **do**
- 8         Update the coefficient  $r_t$  by Eq.(20).
- 9         Update the sparse error  $f_t$  by Eq.(22).
- 10     **end**
- 11     Stage 2: compute  $L_t, U_t$  and  $V_t$
- 12     **while** *not converged* **do**
- 13         Update the additional matrix  $Y_t$  by Eq.(28).
- 14         Update the matrix  $L_t$  by Eq.(30).
- 15         Update the  $U_t$  by Eq.(31).
- 16         Update the  $V_t$  by Eq.(32).
- 17     **end**
- 18 **end**

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## 4. Experimental Results and Discussions

In this section, we investigate the performance of the proposed method. All the experiments were implemented on a PC with an Intel Core i5 CPU at 2.4 GHz and with 16 GB of memory. Simulations were performed using MATLAB 2014a.

Experiments are performed on scene background initialization (SBI) dataset [25]<sup>3</sup>. The SBI dataset, consisting of fourteen image sequences with ground truth images and a set of commonly adopted metrics, with a wide range of complex backgrounds and different situations, including a variety of sequences, such as HallMonitor, Board, CAVIAR1, CaVignal and HumanBody2, and it has been adopted by many existing and new background initialization methods. For the convenience of comparison, the ground truth images of the background are also provided.

<sup>3</sup> <http://sbmi2015.na.icar.cnr.it/>



For detailed qualitative comparisons, the proposed method is evaluated by comparing it with the online RPCA method [32], OMA-RPCA method [40], and OTNNR method [10]. These methods are state-of-the-art techniques for online background modeling.

We implemented the following settings:  $\lambda_1 = 0.1$ ,  $\lambda_2 = 1$ ,  $\lambda_3 = 0.1$ , the convergence error is set to  $10e-4$ , the rank of the low dimensional subspace is 5, the threshold  $\tau$  is set by the average intensity of the optical flow. We implemented these four algorithms on the more challenging moving background video sequences, i.e., HallMonitor, Board, CAVIAR1, CaVignal and HumanBody2. Table 1 describes of the information of datasets.

**Table 1.** Brief introduction of the datasets

Dataset	Size	Number of frames
HallMonitor	352*240	299
Board	200*164	227
CAVIAR1	384*256	609
CaVignal	200*136	257
HumanBody2	320*240	740

#### 4.1. Qualitative Results and Comparison

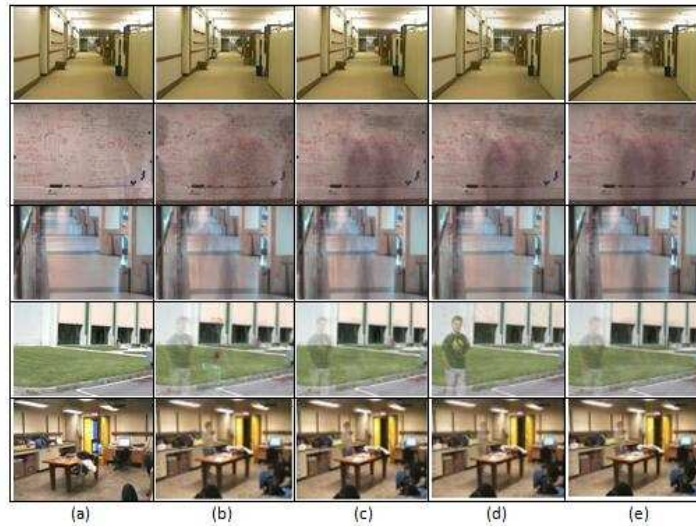
In this paper, we use two methods to get the motion matrix. The first method is to adopt the optical flow [1], and the second method is to apply joint motion detection and frame selection [31]. For these two methods of obtaining the motion matrix, the results of our proposed method and the existing methods are shown in Fig. 1 and Fig. 2 respectively.

Due to the slow moving foreground takes up many areas of the frame, it used to be severely tailed by using the previous methods to recover the true background, such as on-line RPCA, OMA-RPCA and OTNNR. The slow moving foreground may be considered as part of the background. From the results as shown in Fig. 1 and Fig. 2, we can see that our method is the closest to the real background and is superior to other algorithms. Fig. 1 and Fig. 2 show the background modeling results of frame selection mode and optical flow mode respectively. Because of abandoning the redundant frames in the frame selection technology, the effect of background modeling is much better than that of optical flow methods. But for video object with long static foreground, the effect of the two methods are both not very good.

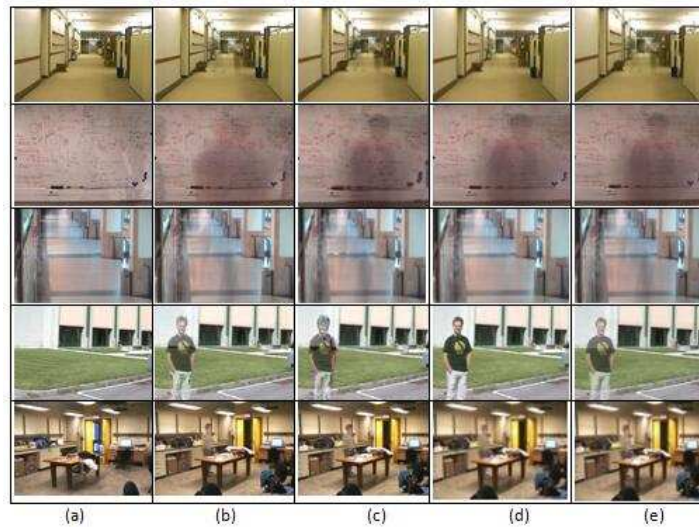
#### 4.2. Quantitative Evaluations and Analysis

In order to quantitatively analyze the performance of the algorithm and verify the rationality of the algorithm, we measured these methods with the PSNR(Peak Signal-to-Noise Ratio) and RRE(Relative Reconstruction Error), which is defined as follows:

$$RRE = \|\hat{A} - A\|_F / \|A\|_F \quad (33)$$



**Fig. 1.** Background modeling results by frame selection method: (a) true backgrounds, (b) OM-RPCAT(Ours), (c) OMA-RPCA, (d) online RPCA, (e) OTNNR. From top to bottom, the recovered backgrounds for HallMonitor, Board, CAVIAR1, CaVignal and HumanBody2 are presented respectively.



**Fig. 2.** Background modeling results by optical flow method:(a) true backgrounds, (b) OM-RPCAT(Ours), (c) OMA-RPCA, (d) online RPCA, (e) OTNNR. From top to bottom, the recovered backgrounds for HallMonitor, Board, CAVIAR1, CaVignal and HumanBody2 are presented, respectively.

$$PSNR = 10 * \log_{10}(255^2 / MSE) \quad (34)$$

where the MSE is the mean square error of the real background and the reconstruction background.

**Table 2.** Quantitative background modeling results by frame selection

	PSNR	RRE
OM-RPCAT(OURS)	83.6	0.014
OMA-RPCA[40]	83.4	0.015
OTNNR[10]	75.8	0.021
Online RPCA[32]	76.1	0.019

**Table 3.** Quantitative background modeling results by optical flow

	PSNR	RRE
OM-RPCAT(OURS)	79.3	0.024
OMA-RPCA[40]	74.1	0.023
OTNNR[10]	75.7	0.027
Online RPCA[32]	76.3	0.020

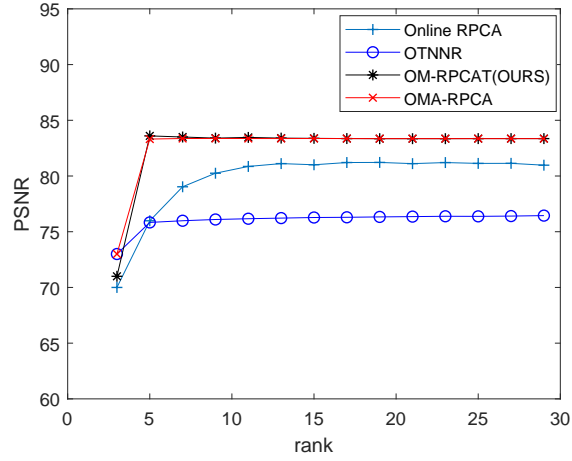
In order to ensure the fairness of verification, we all use the HallMonitor dataset. Based on the results of Table 2 and Table 3, our method can get the best PSNR. In addition, the frame selection method obtain greater performance than optical flow.

### 4.3. Implementation Details and Computational Time

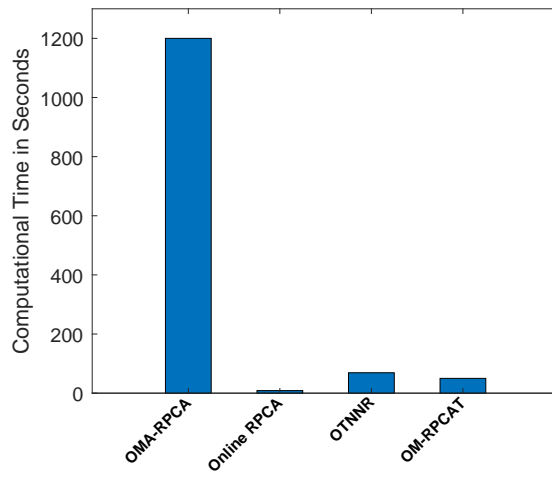
The solution of the proposed models require a set of parameters including  $\lambda_1, \lambda_2, \lambda_3, \tau$  and  $d$ . The  $d$  represents the rank of the low dimensional subspace. In order to update the model quickly, in the previous experiment, we set the rank  $d$  to 5. Now we analyze the performance impact of rank  $d$  on background modeling, HallMonitor dataset was used in this experiment. The frame selection method is selected for the motion estimation.

It can be seen from Fig. 3 that with the change of rank  $d$ , the PSNR also changes. When the  $d$  is set to 5, the performance of background modeling is the best.

In the experiment, we also study the time complexity problem. To compare the overall computational time, we selected a short sequence named HallMonitor. For a fair comparison with the other methods, the time is recorded in seconds. The frame selection method is selected for the motion estimation. Fig. 4 presents the performance in terms of computational time. Compared with the previous algorithms, although our method does not achieve the most promising results, considering the final performance, our algorithm is better.



**Fig. 3.** PSNR analysis of the rank of the low dimensional subspace  $d$



**Fig. 4.** Comparison of computational time in seconds

## 5. Conclusions

In this paper, we propose a novel model for background modeling from a given sequence of video frames and adopt the truncated nuclear norm as the convex optimization framework. In order to perceive motion information in video, we use motion information as the weighting matrix. In addition, the low-rank approximation is optimized to online mode, which is more efficient for the online video. Additionally, to achieve better effects and improve the processing efficiency, we introduced two methods to get the motion estimation matrix. The first method is to adopt the optical flow, and the second method involves joining the motion detection and frame selection which can use the frame selection technique to abandon the redundant frames. We further designed an online optimization scheme to solve the matrix decomposition problem with weighted matrix. Experimental results demonstrate that the proposed algorithm outperforms existing online RPCA algorithms significantly. Considering the limitations of RPCA, our future work will focus on designing more efficient background modeling algorithms.

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**Xu Weiyao** received the master's degree in communication and information systems from Nanjing University of Posts and Telecommunications. He is currently pursuing the Ph.D. degree with the Beijing University of Posts and Telecommunications. His current research interests include around machine learning and human action recognition.

**Xia Ting** received the master's degree in Communication and Information Systems from Hangzhou University of Electronic Science and Technology. She is currently a lecturer at Zaozhuang University. Her current research interests include machine learning and human action recognition.

**Jing Changqiang** received Ph.D. degrees from Kwangwoon University, Korea in 2015. His research interests include computer vision, and wireless sensor network.

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