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Comprehensive Approach to the Design of Information Systems and Optimization of Technical Solutions according to Many Criteria ⋆

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Abstract. One of the problems of the modern information society is the development of effective complex multiprocessor information systems, taking into account the rational use of system resources. The problem of finding the optimal variant of a multiprocessor system is presented as a problem of multicriteria optimization and makes it possible to search for a trade-off between several alternatives. The paper describes several approaches to the comparative assessment of multiprocessor systems, including the search for the non-dominated solutions; narrowing down the Pareto space, using the additional expert information; converting the problem to single-criteria optimization with convolution of criteria; searching for the optimal solution that is closest to the reference point. In the paper the authors propose a ranking method to evaluate technical solutions. The method is based on ranking individual alternatives for each optimization criterion separately, followed by aggregation of ordered ranked lists. The advantage of using the ranking methods is to obtain a complete rating of technical solutions based on their effectiveness, assessed by several criteria. The paper has an educational character and considers the problem of finding a trade-off between system parameters when looking for technical solutions. The practical results of applying different approaches are demonstrated using a simple example.

Keywords: multiprocessor information systems, multicriteria optimization, genetic algorithm, non-dominated solutions, alternatives ranking.

1. Introduction

One of the problems of the modern information society is the development of effective complex multiprocessor information systems, taking into account the rational use of system resources [18]. Examples of such complex systems are global search systems, new

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generation information systems “smart home”, “smart city”, “smart government”, cyber-physical systems of autonomous transport, robotic enterprises, etc. Different types of system resources can be considered, including technical resources - functional complexity, algorithmic complexity, hardware complexity, computing performance, mass-dimensional characteristics, energy consumption, etc.; operational resources - reliability, safety, acceptable service life, etc.; economic resources – cost of the system, cost of maintenance, cost of development support and modifications, etc. It is obvious that most of the resources available for optimization are closely interconnected and their rational distribution during design is only possible using a comprehensive approach.

A comprehensive approach and the search for a trade off between the key parameters of the system for new technical solutions include the following subtasks:

- determining the scope of possible technical solutions when using a given method (or architecture, or concept) of implementation;
- scientific justification of objective limitations in the design of complex systems;
- determination of criteria for choosing technical solutions;
- comparative assessment and ranking of technical solutions:
- identifying trends in the development of complex information systems, etc.

These multi-criteria tasks are difficult to formalize and therefore developers often rely on their experience in solving similar problems or informal methods of expert assessments. From formal system analysis tools, methods of mathematical statistics and operations research are usually used [8],[4]. The determination of the feasible solution set is a major challenge in engineering optimization problems. In multi-criteria problems the researcher can often identify the main variables, establish connections between them, i.e. build a model that adequately reflects the situation, but the preferred combinations of criteria cannot be determined on the basis of objective information. For selection the best solutions a compromise between various criteria is required.

In this paper we observe several approaches to solving the technical problem together with proposing the method of searching for the optimal multi-criteria ranking of alternatives on the basis of individual single-criterion rankings. The proposed research is the extended version of conference paper [18], presented at CERCIRAS 2021 Workshop. Previous paper described the basics of the problem of complex assessments of the technical solutions and the formal approach for solving this problem, including the methods from the theory of Data Mining and Operation Research. It introduced the concept of multicriteria optimization in finding the optimal variant of a multiprocessor system and in searching for a trade-off between several alternatives. The current study extends the research in this direction. Based on a simple example it demonstrates several approaches to searching and narrowing down the Pareto space, including those involving expert knowledge. The approaches for converting a multi-criteria problem to a single-criteria optimization problem by criteria convolution, as well as for ranking solutions based on distances to the ideal point are described and presented on the example. In the last section it is proposed a method based on aggregating ordered ranked lists for searching for the effective technical solutions, characterized by several system resources. The initial rankings of alternatives for each criterion are aggregated in order to find the optimal solution. In this case a single-criteria minimization problem takes into account the sum of distances of the solution candidates to the initial rankings. The proposed method provides more flexibility in selecting the best variant of solution based on several criteria. The method allows to take into account both the objective values of criteria and the decision maker preferences.

The paper notes the advantages and disadvantages of various approaches, as well as the dependence of the result on both the initial parameters of the mathematical models and the expert preferences. It is noted that the use of the methods for ranking solutions allows determining not only the best solution, which is often of primary interest, but a complete rating of all solutions, which provides information for decision-making.

2. Related work

Multi-criteria decision analysis (MCDA) is a multi-step process consisting of a set of methods to structure and formalise decision-making processes in a transparent and consistent manner. Over the years, MCDA methods and software tools are used for a large number of applications from modeling, optimization and decision-making tasks, to performance's simulation [10].

To date, a lot of research has been carried out in the field of multi-criteria decision selection [17]. They are aimed to model decision making process and require the participation of experts in reaching a decision based on many criteria [9]. The most popular are the AHP method, based on pairwise comparison of hierarchical criteria considering difference information; ANP method, which is a non-linear and more general type of AHP using Markov-chain-based aggregation; FUZZY AHP method with the fuzzy evaluation of the alternatives; ELECTE method, based on outranking the relationship of the alternatives and using pairwise comparison; PRAGMA method, which compares partial profiles of alternatives considering all the possible criteria pairs and etc.

In traditional MCDA methods often the single optimal solution is chosen by collecting the DM's preferences where multicriteria optimization (MCO) and decision-making tasks are combined for obtaining a point by point search approach [10]. The final obtained solutions must be as close to the true optimal solution as possible and the solution must satisfy the preference information.

When considering multi-criteria task from the point of view of MCO theory, a non-dominated set in the criteria space or a Pareto-efficient set in the solution space is usually considered. In multicriteria methods, a solution to an MCO problem is understood as a single point of a Pareto-efficient set that is preferable for the decision maker. Although sometimes MCO methods require finding a small number of solutions that are interesting from the decision maker's point of view [6].

Methods for solving the MCO problem in the framework of MCDA are extremely diverse [11] . There are several ways to classify these methods [17]. Considering the decision-making process and the additional information about the preferences of the decision maker the following classes can be distinguished:

- methods that do not take into account the preferences of the decision maker (no-preference methods);
- a posteriori methods;
- a priori methods;
- interactive methods.

In the first class of methods the task is to find some compromise solution, usually in the central part of the Pareto front or the construction of a scalar optimization function without the participation of decision makers. A posteriori methods involve the decision maker entering information about their preferences into the MCO system after a certain set of non-dominated solutions has been obtained. In this regard, all methods of this class at the first stage construct an approximation of the Pareto set. A priori methods are designed to overcome the main disadvantage of a posteriori methods associated with the construction of the entire reachability set. Here it is assumed that the decision maker introduces additional information about his preferences before starting to solve the problem, a priori. Most often, this information is formalized in such a way as to reduce a multi-criteria problem to a single-criteria one. Examples include the scalar convolution method, the e-constraint method [12], lexicographic ordering and goal programming [1]. Interactive methods consist of a set of iterations, each of which includes an analysis stage performed by the decision maker and a calculation stage performed by the MCO system [7]. Based on the nature of the information received by the MCO system from the decision maker at the analysis stage, classes of interactive methods can be distinguished, in which the decision maker directly assigns weighting coefficients to particular optimality criteria; imposes restrictions on the values of particular optimality criteria or evaluates the alternatives proposed by the MCO system. The selection of the appropriate MCO method for new technical solutions depends on the complexity of the decision space, existence and type of expert knowledge and available time. Each MCO method has its own definition of best alternative and it is not determined if using same input data in different methods will give the same results.

Our paper makes the overview and comparative assessment of several a posteriori and a priori approaches to the selection of best variant of technical system based on several criteria, formulating this problem as a MCO task. In the course of describing the methods and applying them using a simple example we demonstrate how similar or different results might evolve. Together with several popular approaches, the method based on aggregation of ordered ranked lists is described. It can be considered as a way of integration between MCO and MCDA processes and shows its consistency on a par with well-known methods. The method allows searching for solution based on both the available numerical criteria values and the expert preferences in ranking alternatives on several criteria.

The described approaches can be applied to the practical tasks in the area of technical system design and can be integrated as the component of a computer system that supports engineering decision-making activities.

3. Comprehensive Approach to System Design of Multiprocessor Information Systems

3.1. Formulation of the System Design Problem

The scope of application of an information system is mainly determined by both functions and technical characteristics, which are specified by numerical or nominal values and limitations. For example, the multiprocessor on-board electronics system of a modern car or robotic mobile platform has a distributed multiprocessor architecture, which allows for real-time control of units (i.e., with a given performance). To provide interprocessor interfaces, a CAN (Controller Area Network) bus is used, which provides sufficient noise immunity and reliability of communications. The number of peripheral processors is a consequence of the number of terminal devices (i.e., the required functional performance). In addition, a consequence of the second order are the parameters of power consumption and mass-dimensional characteristics. If the design domain is a distributed computing system on a chip (SoC), then the priority relationships of mutual influence will be changed and supplemented, perhaps significantly.

The composition and mutual influence of the specified characteristics (and/or parameters) of the system can be represented in the form of an undirected graph, as shown in Fig. 1. In Fig. 1, the graph reflects only qualitative dependencies. Even a quick analysis of a fragment of the original graph indicates the close interdependence of parameters in the system. Sometimes the named dependencies are obvious, for example, performance versus the number of processors, and they are easy to represent in the form of formulas. However, this is not always the case. For example, it is difficult (and perhaps impossible) to indicate the dependence of the reliability or mechanical layout (weight and dimensions) of a microprocessor system on the selected type of interface, although it is obvious to every designer that such a dependence exists. This graph is for demonstration purposes and, naturally, can be supplemented with other parameters and connections.

In general, the task of system design is to find a conceptual solution whose parameters will be within acceptable, or better yet, optimal values.

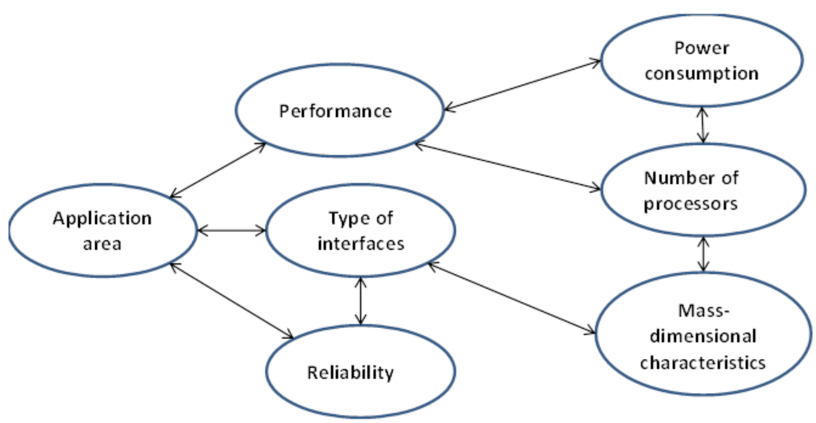

Fig. 1. Undirected graph, reflecting the subject area of system design

3.2. System Design Methodology

The proposed system design methodology is as follows:

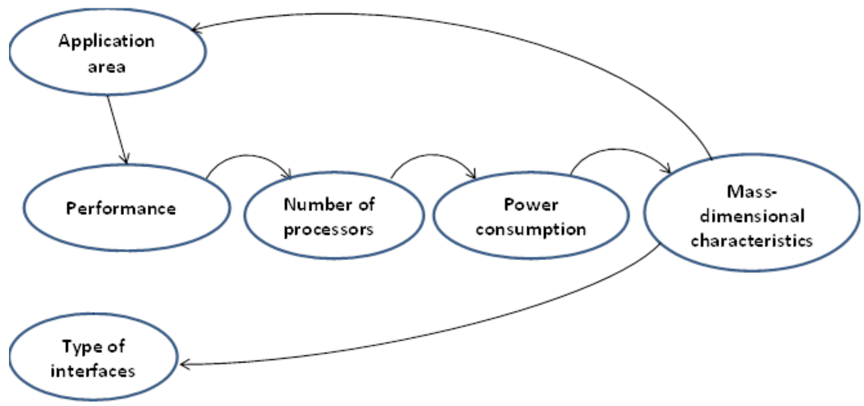

Fig. 2. A directed graph reflects one of the possible system design scenarios

1. The developer (or general designer, or system architect) draws up an initial graph, similar to Fig. 1, reflecting two-way relationships in system design in a given subject area.
2. In the undirected graph, the expert identifies key and derived parameters (the reliability parameter is not considered as a key one).
3. The original graph is modified into a directed one, and the number of mutual (less significant) connections is reduced. Ideally, there could be a chain of cause and effect relationships. In Fig. 2 on the basis of expert experience and understanding of all the restrictions available to him, a special case of relationships are considered. According to Fig. 2, to ensure a given performance it is necessary to use, for example, 6 processors, which in turn will increase energy consumption by 6 times compared to a single-processor implementation. These changes will cause an increase in weight and size characteristics and will affect the appearance of the interface.
4. The found chain is supplemented (detailed) with the necessary (known) quantitative data and restrictions. A rapid evaluation of the possibility of achieving the result is carried out using known theoretical dependencies, expert knowledge, as well as multi-criteria data analysis.
5. Very often parameters have implicit and mutually exclusive relationships. Therefore, it is important to “check feedback” during system design. For example, it may turn out that the obtained mass-dimensional characteristics are not consistent with the conceptually accepted interface, or the subject area. In this case, the system design process is iteratively repeated until an acceptable system solution is found.

4. Comparative Evaluation of Multiprocessor Systems as a Problem of Multicriteria Optimization

The greatest difficulty in designing complex multiprocessor information systems lies in the need to solve the problem of multicriteria optimization according to multiple optimization criteria, which are individual system parameters or used system resources. As a rule, the criteria are interdependent, i.e. an increase in one of them can lead to a decrease in the value for the other. Using multicriteria optimization makes it possible to identify compromise or non-dominant solutions, where none of them is better than the other in all the parameters under consideration and, therefore, are of equal importance. In this case, many solutions are allowed, each of which is acceptable in the absence of preliminary information about the importance of the criteria.

In general, the problem of multicriteria optimization is formulated as follows: Find the vector $\bar{x}^* = (x_1^*, x_2^*, \ldots x_n^*)^T$ of the values of parameters, which satisfy m inequalities:

$$g_i(\bar{x}) \geq 0,\ i = 1, 2, \ldots, m, \quad (1)$$

and p equalities

$$h_i(\bar{x}) = 0,\ i = 1, 2, \ldots, p, \quad (2)$$

and optimize the vector function

$$f(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), \ldots, f_k(\bar{x})]^T\,. \quad (3)$$

Constraints (1), (2) define the domain G, which contains all feasible solutions to the problem. The vector $\bar{x}^*$ corresponds to the optimal solution in the domain G. In this case, the optimality is meant according to the Pareto concept, the formal definition of which from the point of view of the maximization problem is as follows [3]:

The vector of solution $\bar{x}^*$ is called Pareto-optimal if and only if there is no other vector $\bar{x}$,which dominate $\bar{x}^*$,i.e. if

$$\forall i \in 1, 2, \ldots, k,\ f_i(\bar{x}) \leq f_i(\bar{x}^*)\ and\ \exists i \in 1, 2, \ldots, k,\ where\ f_i(\bar{x}) < f_i(\bar{x}^*)$$

In other words, $\bar{x}^*$ is Pareto optimal if there are no acceptable vectors $\bar{x}$, which allow you to increase the value of one of the criteria, while not decreasing the value of at least one of the remaining criteria. The solution $\bar{x}^*$ is strictly dominates the solution $\bar{x}$, if $\forall i \in 1, 2, \ldots, k,\ f_i(\bar{x}) < f_i(\bar{x}^*)$. In general, the solution to the optimization problem is a set of non-dominated solutions. Having a set of several non-dominated solutions obtained as a result of multicriteria optimization, it is possible to choose a solution that is most preferable for a specific applied problem.

Consider a demonstration example of solving the problem of comparative evaluation of multiprocessor systems. Table 1 presents four multiprocessor systems, which are characterized by several parameters: the number of processors, performance, dimensions, power consumption. We will assume that these systems can programmatically solve the same problem, but are implemented in a different design.

For clarity and simplicity of reasoning, we use the simplest normalization method, when the parameters are reduced to some fixed maximum value (in our case, 100, 10, 10 and 100, respectively). The normalized values of the parameters of multiprocessor systems are shown in Table 1 in the corresponding columns after the sign (/).

Table 1. Parameters of multiprocessor systems

System	Number of processors (items)	Performance (Gflops)	Dimensions (Volume, dm3)	Power consumption (W)
System 1	4/0,04	2,0/0,2	4,0/0,4	20,0/0,2
System 2	8/0,08	3,0/0,3	1,0/0,1	12,0/0,12
System 3	16/0,16	5,0/0,5	3,0/0,3	10,0/0,1
System 4	64/0,64	7,0/0,7	6,0/0,6	40,0/0,4

If the number of processors and performance are known, statistically confirmed parametric dependencies (Fig. 3a), then the dependences of dimensions and power consumption in this example are not visible (Fig. 3b). (Although, in theory, we know that with an increase in the number of processors and performance, the power consumption and size should increase). But this example deliberately does not provide any information about the technology, element base, design, purpose, generation, etc., which directly affects both the dimensions and power consumption of the implemented systems.

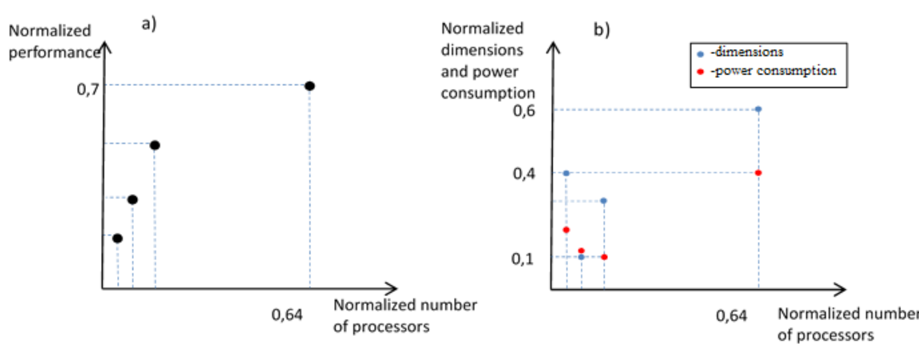

Fig. 3. Representation of multiprocessor systems in two-dimensional parameter space

Let perform the assessment of the systems presented in Table 1 based on the concept of non-dominated solutions, considering the value of each criterion separately. We will assume that it makes no sense to include the number of processors as an individual criterion, since it is indirectly in the performance of systems. In our case we will perform a manual assessment of the systems. To solve more complex problems with a large number of alternatives, characterized by a large number of parameters, special methods of multicriteria optimization are used [15], [3],[19].

In the presented example, the criteria to be optimized correspond to the parameters of the system, and the problem of multicriteria optimization in this case can be written in the following form:

Optimize vector function

$$F(x) = [x_1, x_2, x_3]$$
$$x_1 \to max, x_2 \to min,\ x_3 \to min \tag{4}$$

and $a_1 \le x_1 \le b_1, a_2 \le x_2 \le b_2, a_3 \le x_3 \le b_3$ – allowed parameter range, where x_1, x_2, x_3 - performance, dimensions and power consumption.

Consider Systems 1 and 2, where the value of the performance criterion of System 2 are higher than the corresponding value of System 1, and the value of the dimensions and power consumption criteria for System 2 are lower. Consequently, System 2 has more optimal parameter values and thus dominates System 1. If we compare Systems 2 and 3, then we see that System 3 has higher performance and less power consumption, while its dimensions are larger than the dimensions of System 2. Therefore, Systems 2 and 3 are non-dominated solutions. Likewise, the non-dominated solutions are Systems 2 and 4, Systems 3 and 4.

In Fig. 4 the systems under consideration are shown as points in the space of two criteria. Non-dominated solutions are connected with a line.

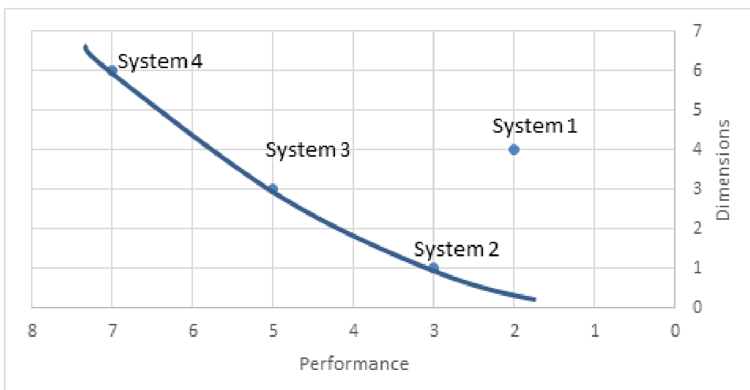

Fig. 4. Graphic presentation of technical solutions in a two-criteria space

It should be noted that Systems 2, 3, 4 are not dominant in the specific problem under consideration, i.e. they are locally non-dominated solutions. In the case of adding additional alternative solutions for multiprocessor systems, the number and composition of non-dominated solutions may change, although in this case the solutions will be locally non-dominated. According to the analysis System 1 can be excluded from further consideration due to the fact that it is the dominant solution for all the considered optimization criteria.

To further narrow down the Pareto set and identify the single best solution, some additional information is needed [14]. One of the main types of additional information, often used when solving various multicriteria problems, is information about the comparative importance of partial criteria, which is usually given in the form of numerical coefficients $w_k \ge 0$, characterizing the importance of partial criterion $f_i, i = 1, \ldots, k$. The importance coefficients together constitute a weight vector $w = (w_1, \ldots, w_k)$, the components of which are usually normalized by the condition $\sum_k w_k = 1$. The most well-known methods for calculating coefficients include sequential comparison of criteria by importance, pairwise comparison of criteria by absolute or relative importance.

Using additional information about the importance of criteria, you can narrow down the set of non-dominated solutions. Consider two non-dominated solutions $y_1 = (y_{11}, \ldots, y_{1k})$ and $y_2 = (y_{21}, \ldots, y_{2k})$, where $y_{1p} > y_{2p}, y_{1q} < y_{2q}$ and $y_{1h} = y_{2h}$ for all $h \neq p, q$. Formally, these alternatives are incomparable in terms of dominance. However, for the decision maker (DM), alternative y_1 is preferable to y_2. This means that the p-th partial criterion is more important than the q-th partial criterion with parameters $r_p = y_{1p} - y_{2p} > 0, r_q = y_{2q} - y_{1q} > 0$. When choosing one of two solutions, the DM agrees to lose the value r_q according to a less important criterion in order to receive an additional gain r_p according to a more important criterion.

The number $t_{pq} = \frac{r_q}{(r_p + r_q)}$ is called the proportional coefficient of relative importance for the p-th and q-th criteria. At $t_{pq} = 0,5$, the values of losses and gains coincide. Thus, the set of selected solutions is contained in the restricted Pareto boundary, consisting of vectors $y' = f'(x) = (f'_1(x), \ldots, f'_k(x))$ with components determined by the expressions:

$$f'_q(x) = t_{pq} f_p(x) + (1 - t_{pq}) f_q(x);\ f'_h(x) = f_h(x), \forall h \neq q \tag{5}$$

According to (5), new values of the criteria vector are obtained from the previous ones by replacing the less important criterion $f_q(x)$ with a convex combination of criteria $f_p(x)$ and $f_q(x)$.

Consider the application of the above-mentioned method of narrowing the Pareto space using the example described in Table 1. Let us consider three previously obtained non-dominated solutions, represented by vectors of three criteria values $S_2 = (0,3; -0,1; -0,12)$, $S_3 = (0,5; -0,3; -0,1)$ and $S_4 = (0,7; -0,6; -0,4)$, where the values of the last two criteria are inverted in order to bring them to the maximization problem.

Let us assume that the DM considers the first criterion f_1 more important than the second f_2 with a proportional coefficient of relative importance $t_{12} = 0,8$. Recalculate the second component of each vector $S_i, i = 2, \ldots, 4$ using the formula $f'_2(S_i) = 0,8 f_1(S_i) + 0,2 f_2(S_i)$. We obtain new vectors $S'_2 = (0,3; 0,22; -0,12)$, $S'_3 = (0,5; 0,34; -0,1)$ and $S'_4 = (0,7; 0,44; -0,4)$. It is obvious that solution S'_3 dominates solution S'_2. Therefore, in the narrowed Pareto space there will remain two solutions S'_3 and S'_4, which correspond to System 3 and System 4.

5. Approaches to Converting a Multi-Criteria Problem to a Single-Criteria One

One of the ways to solve a multi-criteria problem for selection a technical solution is to transform it into a single-criteria one by combining all partial criteria $f_j(x), j = 1, \ldots, k$ into one general quality criterion $f(x) = F(f_1(x), f_2(x), \ldots, f_k(x))$, which is otherwise called criteria convolution. Then the search for the best solution reduces to finding the extremum of the single function $f(x)$

$$x^* \in argmax_{x \in X^a} f(x), \tag{6}$$

where x is an alternative from the acceptable set X^a.

As a rule, weighted convolutions of partial criteria for the effectiveness of a technical solution are used

$$f(x) = \sum_{j=1}^{k} w_j f_j(x)$$
$$f(x) = \prod_{j=1}^{k} w_j f_j(x) \; f(x) = \prod_{j=1}^{k} [f_j(x)]^{w_j}, \tag{7}$$

where $w_j \geq 0$ is a weight of the partial criterion $f_j(x)$.

Partial performance criteria are usually normalized in one of the following ways:

$$f'_j(x) = \frac{f_j(x)}{y_j^{max}},\; f'_j(x) = \frac{f_j(x)}{(y_j^{max} - y_j^{min})},\; f'_j(x) = \frac{f_j(x) - y_j^{min}}{(y_j^{max} - y_j^{min})} \tag{8}$$

where y_j^{min}, y_j^{max} – minimal and maximal values of the partial criterion $f_j(x)$.

The choice of the weights of the partial criteria in the optimization function (7) can influence the result of the optimal solution and is usually set by the DM by a number of well-known methods [13]. Consider the implementation of the above approach for the example from Table 1. The complex criterion for the comparative assessment of systems is a weighted sum of three criteria. In this case, it is necessary to perform the following steps:

1. Bringing the criteria to a single range of values (normalization or scaling of the criteria):
$$\bar{x_i} = \frac{x_i - x_i^{min}}{x_i^{max} - x_i^{min}},$$
where x_i^{min} is the minimal value of i-th criterion, x_i^{max} is the maximal value of i-th criterion. For our example $x_i^{min} = 0$ for $i = 1, 2, 3$ and $x_1^{max} = 10$, $x_2^{max} = 10$, $x_3^{max} = 100$.
2. Selection of weight coefficients w_1, w_2, w_3 for each criterion: performance, dimensions and power consumption.
3. Ranking solutions according to a complex criterion $F(x) = -w_1\bar{x_1} + w_2\bar{x_2} + w_3\bar{x_3}$, where the value of w_1 is taken with minus sign to convert to the minimization problem according to the first criterion.

The minimum value of the complex criterion corresponds to the most optimal solution. The normalized values of the system parameters are presented in Table 1. Let us choose the following values of the weight coefficients $w_1 = w_2 = w_3 = 1$. Let's calculate the values of the complex criterion for each of the systems $S_1 = (0,2; 0,4; 0,2)$, $S_2 = (0,3; 0,1; 0,12)$, $S_3 = (0,5; 0,3; 0,1)$ and $S_4 = (0,7; 0,6; 0,4)$

$$F_1(x) = -1 * 0,2 + 1 * 0,4 + 1 * 0,2 = 0,4$$
$$F_2(x) = -1 * 0,3 + 1 * 0,1 + 1 * 0,12 = -0,08$$
$$F_3(x) = -1 * 0,5 + 1 * 0,3 + 1 * 0,1 = -0,1$$
$$F_4(x) = -1 * 0,7 + 1 * 0,6 + 1 * 0,4 = 0,3.$$

Thus, the systems presented in Table 1 according to the complex criterion can be ranked as follows System 3, System 2, System 4 and System 1 in ascending order of the criterion value. The most optimal system is System 3.

The disadvantage of this ranking method is the dependence of the optimal solution on the choice of the values of the weight coefficients w_1, w_2, w_3. For example, if the value of the weight $w_1 = 0.5$, then System 2 will be selected as the most optimal (Fig. 5).

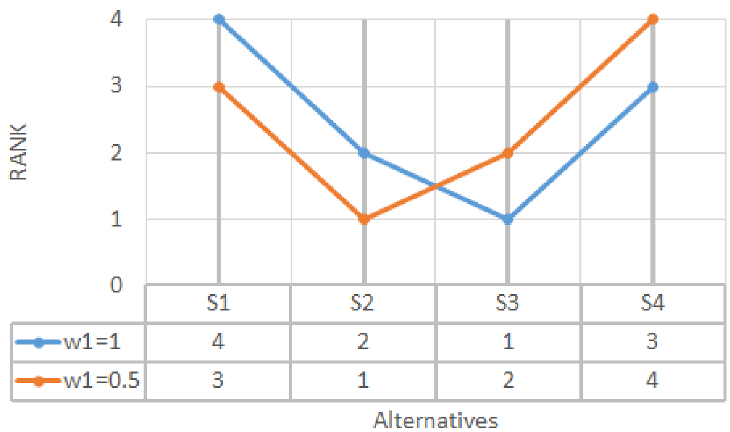

Fig. 5. Ranking of alternatives according to a complex criterion $F(x)$ with different values of weight coefficient w_1

Another approach for solving a multicriteria problem is to search for alternatives with given characteristics. In this case, you can indicate the values of partial quality criteria $y_j^0, j = 1, \ldots, k$ that are desirable for the DM or the boundaries of their change. The set of such values $y = (y_1^0, \ldots, y_k^0)$ is called the reference point. Two characteristic reference points are, in particular, $y^{max} = (y_1^{max}, \ldots, y_k^{max})$ and $y^{min} = (y_1^{min}, \ldots, y_k^{min})$, where y_i^{min} is the minimal value of i-th criterion, y_i^{max} is the maximal value of i-th criterion.

To solve a multicriteria optimization problem, a search is made for an alternative or technical solution that is closest to the reference point. In the multidimensional space of evaluations based on partial quality criteria, a certain proximity measure $d[f(x), y^0]$ is specified between the points $f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$ and $y^0 = (y_1^0, \ldots, y_k^0)$, where $f(x)$ is the alternative and y^0 is the reference point. Then the optimal solution is defined as

$$x^* \in argmin_{x \in X^a} d[\, f(x), y^0] \tag{9}$$

Usually one of the metrics of the k-dimensional vector space (R^k, d_p) is chosen as a proximity measure, such as the weighted Euclidean metric

$$d_2[f(x), y^0] = [\sum_{j=1}^{m} w_j (f_j(x) - y_j^0)^2]^{\frac{1}{2}}, \tag{10}$$

where w_j is the coefficient of importance of the partial criterion $f_j(x)$.

Setting one or another proximity measure $d[f(x), y^0]$ is another possible way of convolving partial criteria and transforming a multicriteria problem into a single-criteria one. The disadvantage is that different metrics $d[f(x), y^0]$ may correspond to different optimal options for the technical solution x^*. Despite the disadvantage, this method for solving a multicriteria problem is widely used in practice [13].

One example of the implementation of multicriteria optimization taking into account reference points is the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method proposed in [5]. The main idea of the method is as follows: after determining the "ideal" or best and "ideal-negative" or worst expected states (alternatives), an attempt is made to find a solution that would allow one to get as close as possible to the "ideal" state and remain as far away from the "ideal-negative" state. The decision-making process begins with the evaluation of all alternative solutions according to all criteria. As a result, a decision matrix is formed. The method consists of six consecutive steps: 1) calculation of the normalized decision matrix; 2) calculation of a weighted normalized decision matrix; 3) definition of the "ideal" and "ideal-negative" expected state; 4) calculation of the metric values; 5) calculation of relative proximity to the "ideal" state; 6) ranking of alternatives.

Mathematical description of the TOPSIS method is as follows:

Step 1. Formation of a matrix of assessments or decisions $(x_{ij})_{n\times k}$, consisting of n alternatives and k criteria, where the element of the matrix x_{ij} determines the value of the j-th criterion of the i-th alternative.

Step 2. By normalizing the matrix $(x_{ij})_{n\times k}$ forming the matrix $R = (r_{ij})_{n\times k}$ as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{t=1}^{n} x_{tj}^2}},\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, k. \tag{11}$$

Step 3. Calculation of the weighted normalized decision matrix:

$$t_{ij} = r_{ij} \cdot w_j,\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, k, \tag{12}$$

where w_j – weight coefficient of j-th criterion, $j = 1, 2, \ldots, k$ and $\sum_{j=1}^{k} w_j = 1$.

Step 4. Determination of the worst A_w and best A_b alternative:

$$\begin{aligned} A_w &= \{\langle max(t_{ij}|i = 1, 2, \ldots, n)|j \in J_-\rangle, \\ &\quad \langle min(t_{ij}|i = 1, 2, \ldots, n)|j \in J_+\rangle\} \equiv \{t_{wj}|j = 1, 2, \ldots, k\}, \\ A_b &= \{\langle min(t_{ij}|i = 1, 2, \ldots, n)|j \in J_-\rangle, \\ &\quad \langle max(t_{ij}|i = 1, 2, \ldots, n)|j \in J_+\rangle\} \equiv \{t_{bj}|j = 1, 2, \ldots, k\}, \end{aligned} \tag{13}$$

According to (13) the value of an individual criterion of the worst alternative A_w is equal to the maximum value of this criterion for the alternatives under consideration in the case of minimizing the criterion ($j \in J_-$) and equal to the minimum value of this criterion for the alternatives under consideration in the case of maximizing the criterion ($j \in J_+$). And the reverse reasoning applies to the best alternative A_b.

Step 5. Calculation of L^2-distance between target alternative i and the worst alternative A_w

$$d_{iw} = \sqrt{\sum_{j=1}^{k} (t_{ij} - t_{wj})^2},\ i = 1, 2, \ldots, n, \tag{14}$$

and the distance between i-th alternative and the best alternative A_b

$$d_{ib} = \sqrt{\sum_{j=1}^{k}(t_{ij} - t_{bj})^2}, \ i = 1, 2, \ldots, n, \tag{15}$$

where d_{iw} and d_{ib} are distances according to the L^2-norm.

Step 6. Calculation of relative proximity to the ideal solution:

$$s_{iw} = \frac{d_{iw}}{(d_{iw} + d_{ib})}, \ 0 \leq s_{iw} \leq 1, \ i = 1, 2, \ldots, n. \tag{16}$$

The value $s_{iw} = 1$ if and only if the alternative corresponds to the best state; and $s_{iw} = 0$ if and only if the alternative corresponds to the worst state.

Step 7. Ranking alternatives by values s_{iw} $(i = 1, 2, \ldots, n)$.

Let's consider the implementation of the TOPSIS method for the example from Table 1. Let us choose the following values of the weight coefficients $w_1 = w_2 = w_3 = 1$. According to (12) the values of the weighted normalized decision matrix $t_{ij}, \ i = 1, 2, 3, 4; \ j = 1, 2, 3$ for alternatives $S_i = t_{ij}, \ i = 1, 2, 3, 4$ are the following $S_1 = (0,2; 0,4; 0,2)$, $S_2 = (0,3; 0,1; 0,12)$, $S_3 = (0,5; 0,3; 0,1)$ and $S_4 = (0,7; 0,6; 0,4)$. According to (13) we define the best alternative as

$$A_b = (\max(0,2; 0,3; 0,5; 0,7); \min(0,4; 0,1; 0,3; 0,6); \min(0,2; 0,12; 0,1; 0,4)) = (0.7; 0.1; 0.1)$$

and the worst alternative as

$$A_w = (\min(0,2; 0,3; 0,5; 0,7); \max(0,4; 0,1; 0,3; 0,6); \max(0,2; 0,12; 0,1; 0,4)) = (0,2; 0,6; 0,4).$$

The graphical representation of the choice of the best A_b and worst A_w alternatives is shown in Fig. 6.

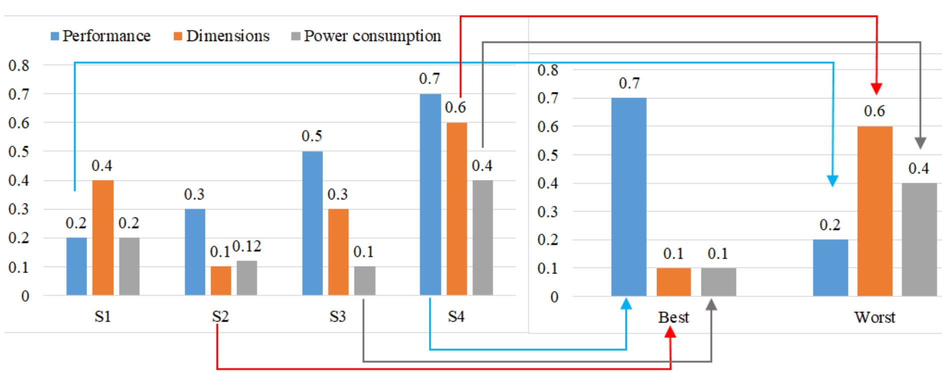

Fig. 6. Illustration of the selection of the best and worst alternatives

According to (14) and (15) L^2-distances between alternatives $S_i,\ i = 1, 2, 3, 4$ and the worst and best alternatives A_w and A_b are

$$d_{1b} = \sqrt{(0,2 - 0.7)^2 + (0,4 - 0.1)^2 + (0,2 - 0.1)^2} = \sqrt{0,25 + 0,09 + 0,01} = \sqrt{0,35} \approx 0,2828$$
$$d_{1w} = \sqrt{(0,2 - 0,2)^2 + (0,4 - 0,6)^2 + (0,2 - 0,4)^2} = \sqrt{0 + 0,04 + 0,04} = \sqrt{0,08} \approx 0,5916$$

$$d_{2b} = \sqrt{(0,3 - 0.7)^2 + (0,1 - 0.1)^2 + (0,12 - 0.1)^2} = \sqrt{0,16 + 0 + 0,0004} = \sqrt{0,1604} \approx 0,4005$$
$$d_{2w} = \sqrt{(0,3 - 0,2)^2 + (0,1 - 0,6)^2 + (0,12 - 0,4)^2} = \sqrt{0,01 + 0,25 + 0,0784} = \sqrt{0,3384} \approx 0,5817$$

$$d_{3b} = \sqrt{(0,5 - 0.7)^2 + (0,3 - 0.1)^2 + (0,1 - 0.1)^2} = \sqrt{0,04 + 0,04 + 0} = \sqrt{0,08} \approx 0,2828$$
$$d_{3w} = \sqrt{(0,5 - 0,2)^2 + (0,3 - 0,6)^2 + (0,1 - 0,4)^2} = \sqrt{0,09 + 0,09 + 0,09} = \sqrt{0,27} \approx 0,5196$$

$$d_{4b} = \sqrt{(0,7 - 0.7)^2 + (0,6 - 0.1)^2 + (0,4 - 0.1)^2} = \sqrt{0 + 0,25 + 0,09} = \sqrt{0,34} \approx 0,5831$$
$$d_{4w} = \sqrt{(0,7 - 0,2)^2 + (0,6 - 0,6)^2 + (0,4 - 0,4)^2} = \sqrt{0,25 + 0 + 0} = \sqrt{0,25} \approx 0,5$$

After determining the distances from each alternative to the positive and negative ideal solutions, the values of relative proximity to the ideal solution s_{iw} are calculated according to (16) and presented in Table 2. According to the values of the relative similarity s_{iw} in Table 2 the systems are ranked as in the last column and the most optimal system is System 3.

Table 2. Alternatives ranking results

Alternative	d_{iw}	d_{ib}	s_{iw}	Rank
S_1	0,2828	0,5916	0,3234	4
S_2	0,5817	0,4005	0,5922	2
S_3	0,5196	0,2828	0,6475	1
S_4	0,5	0,5831	0,4616	3

6. Ranking Technical Solutions by Aggregating Ordered Ranked Lists

In addition to known methods in our study we consider another approach for converting a multicriteria optimization problem to a single-criteria representation when searching for technical solutions. The method is based on ranking individual alternatives for each optimization criterion separately, followed by aggregation of ordered ranked lists [2],[16].

Let there be n alternatives $S_i,\ i = 1, \ldots, n$, each of which is characterized by the values of k indicators (criteria) $S_i = (x_{i1}, \ldots, x_{ik})$, and the indicators can be both quantitative and qualitative. The values of the criteria belong to the set $X^a \subseteq X = X_1 \times \cdots \times X_k$ of acceptable values. Heterogeneous values of indicators are usually transformed to a single measurement scale.

According to the method, all alternatives are initially ranked for each of the k criteria, where L_i is a list of alternatives ordered relative to the values of the i-th criterion. Thus, the following single-criteria minimization problem is formulated:

$$\delta^* = argmin\ \Phi(\delta), \tag{17}$$

where $\Phi(\delta) = \sum_i d(\delta, L_i)$, d is the distance function, and minimization is carried out with respect to all possible ordered lists δ of dimension $n = |L_i|$.

Selecting a suitable distance function d is one of the most important steps of the method. Two options were chosen as such functions: Spearman distance and Kendall rank distance. The Spearman distance between an ordered list of alternatives L_i and any ordered list δ can be defined as

$$S(\delta, L_i) = \sum_{t \in L_i \cup \delta} |r^{\delta}(t) - r^{L_i}(t)|, \tag{18}$$

where $r^{L_i}(t)$ – rank of alternative t in the list L_i.

The smaller the value of the metric $S(\delta, L_i)$, the more similar the two lists. To take into account additional information about the values of the criteria for each alternative a weighted Spearman distance is determined

$$WS(\delta, L_i) = \sum_{t \in L_i \cup \delta} |M(r^{\delta}(t)) - M(r^{L_i}(t))| \cdot |r^{\delta}(t) - r^{L_i}(t)|, \tag{19}$$

where $M(A)$ – value of the i-th criterion of alternative A.

Kendall's rank distance is a metric that counts the number of pairwise divergences between two ranked sets of feature values. The greater this distance, the more different the two characteristics are, and, therefore, the less the dependence between them.

In the case when the number of alternatives is small, the single-criteria problem (17) can be solved by simply searching through the variants of ordered lists and choosing the alternative located at the top position of the list as the optimal solution.

So, for example from Table 1, we can create three ordered lists according to three optimization criteria (Table 3), where the last column presents the optimal ranking δ corresponding to the minimum of the functional $\Phi(\delta) = 8$.

In order to select the optimal ranking δ, we have applied the simple strategy of calculating the $\Phi(\delta)$ value for all possible permutations of the rankings of alternatives $S_i,\ i =$

Table 3. Ranked lists of alternatives

Alternative	Performance	Dimensions	Power consumption	Optimal rank
S_1	4	3	3	3
S_2	3	1	2	2
S_3	2	2	1	1
S_4	1	4	4	4

$1, 2, 3, 4$ from Table 1. After that the ranking δ corresponding to the minimum value of the function $\Phi(\delta)$ is considered as optimal. The $\Phi(\delta)$ values for each ranking of alternatives are presented in Table 4.

Table 4. Functional values for all possible rankings

No	Ranking	Value	No	Ranking	Value
1	(1 2 3 4)	16	**13**	**(3 1 2 4)**	**8**
2	(1 2 4 3)	20	14	(3 1 4 2)	16
3	(1 3 2 4)	14	**15**	**(3 2 1 4)**	**8**
4	(1 3 4 2)	22	16	(3 2 4 1)	16
5	(1 4 2 3)	18	17	(3 4 1 2)	14
6	(1 4 3 2)	22	18	(3 4 2 1)	14
7	(2 1 3 4)	14	19	(4 1 2 3)	10
8	(2 1 4 3)	18	20	(4 1 3 2)	14
9	(2 3 1 4)	12	21	(4 2 1 3)	10
10	(2 3 4 1)	20	22	(4 2 3 1)	14
11	(2 4 1 3)	16	23	(4 3 1 2)	12
12	(2 4 3 1)	20	24	(4 3 2 1)	12

In Table 4 the values in the column "Value" is calculated as

$$\Phi(\delta) = S(\delta, L_1) + S(\delta, L_2) + S(\delta, L_3), \tag{20}$$

where $L_i,\ i = 1, 2, 3$ are the rankings of alternatives for criteria "Performance", "Dimensions" and "Power consumption" respectively, and $S(\delta, L_i)$ is a Spearman distance in (18). For example for ranking No. 1 in Table 4 and using single-criteria rankings in Table 3 the Spearman distances are calculated as

$$S(\delta, L_1) = |1-4| + |2-3| + |3-2| + |4-1| = 3+1+1+3 = 8$$
$$S(\delta, L_2) = |1-3| + |2-1| + |3-2| + |4-4| = 2+1+1+0 = 4$$
$$S(\delta, L_3) = |1-3| + |2-2| + |3-1| + |4-4| = 2+0+2+0 = 4$$

and the value of function in (20) is $\Phi(\delta) = 8 + 4 + 4 = 16$.

In Table 4 the minimal functional value $\Phi(\delta) = 8$ corresponds to the ranking No. 15: System 3, System 2, System 1 and System 4. The most optimal system is System 3. The same value of functional $\Phi(\delta)$ also corresponds to the ranking No.13: System 3, System 1, System 2 and System 4. In case of ambiguity, it makes sense to take into account the real criteria values and estimate the optimal ranking according to (19).

Using Multidimensional Scaling (MDS) to reduce the dimensionality of data the two-dimensional representation of all possible rankings from Table 4 and single-criteria rankings from Table 3 is shown in Fig. 7. In Fig. 7 three single-criteria rankings are marked with red circles and it is clearly seen that the rankings No.15 and No.13, marked in bold have a minimum total distance to three single-criteria rankings compared to other alternatives.

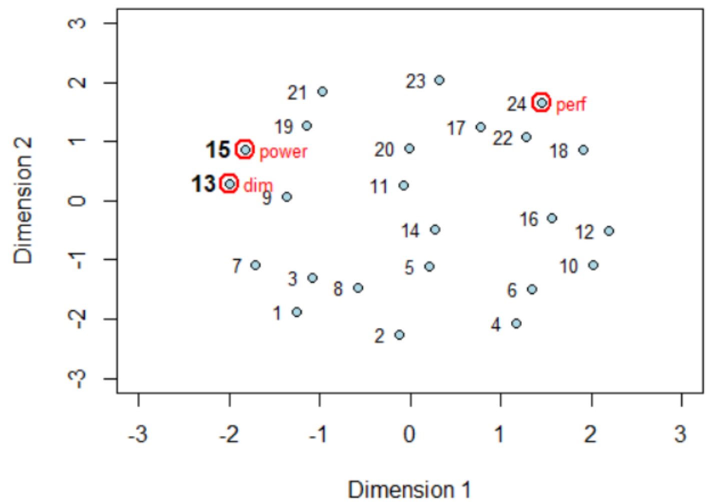

Fig. 7. Graphical representation of all alternative rankings highlighting the optimal ones

In the case of a larger number of alternatives and criteria, it is proposed to solve the optimization problem (17) using a genetic algorithm (GA) [15].

The GA consists of the following steps:

Step 1. $popSize$ ordered lists of dimension n (the number of alternatives) are randomly initialized, which form the initial set of possible solutions to the problem. The size of the population must be proportional to the number of alternatives and the number of unique elements in the original ordered lists $L_i,\ i = 1, 2, \ldots, k$.

Step 2. Depending on which distance is used, calculate the objective function for each element of the population. Then randomly select individuals from the population for the next generation of GA using weighted random sampling, where the weights are determined according to the values of the fitness (objective) function.

Step 3. Carrying out crossover operations with probability p_{cross} (transition probability), i.e. two randomly ordered lists can exchange their parts, which start from a random position with probability p_{cross}.

Step 4. The crossover operation only allows the mixing of ordered lists, but to obtain radically new solutions it is necessary to use mutations. Mutation operations are performed with probability p_{mut} (mutation probability). Thus, any list in a population can randomly change one or more of its elements.

Step 5. The algorithm stops if the optimal list does not change for successive several generations of the GA or the maximum number of iterations (GA epochs) is reached.

The advantage of using the described method to solve the problem of selecting the optimal technical solution is to obtain a complete rating of technical solutions based on their effectiveness, assessed by several criteria. Any number of alternatives with arbitrary combinations of GA parameters and/or distance functions can be considered. In addition, the researcher can decide how many performance criteria to use to obtain a reliable assessment of solutions.

7. Conclusion

This paper presents the results of extended research in the field of optimization of technical solutions according to many criteria. The problem statement and system design methodology are described in general terms. The problem of finding a technical solution, which is characterized by various parameters or used system resources, is presented as a multicriteria optimization problem that allows one to find Pareto-optimal solutions. An approach for narrowing the solution space in order to reduce the uncertainty associated with multi-criteria selection and find the optimal technical solution is described and demonstrated by example. The approaches for converting a multi-criteria selection problem to a single-criteria optimization problem by criteria convolution, as well as for ranking solutions based on calculating distances to the ideal point are described and presented on the example. It is proposed to apply a method based on aggregating ordered ranked lists for converting a multicriteria optimization problem to a single-criteria representation when searching for technical solutions. A method is based on solving a single-criteria optimization problem, where the optimization functional estimates the sum of the distances of the searched optimal ranking of alternatives to the corresponding rankings for each of the criteria under consideration.

The approaches described in the paper can be used to find a compromise between various characteristics of a technical solution in order to select the optimal variant when designing multiprocessor information systems. As a result of applying methods for ranking solutions, it will be possible to determine not only the best solution, which is often of primary interest, but a complete rating of all solutions, which provides information for further research. The practical results of applying different approaches are demonstrated using a simple example.A further direction of research is the analysis and application of interactive methods for searching for the optimal technical solution when developing a multiprocessor information system.

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